Comparative Analysis of Classical and Modified Moses Test

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Abstract

Moses test is a nonparametric method to test the equality of two dispersion parameters. The Moses test does not assume equality of location parameters, and this fact gives the test wider applicability. However, this test is inefficient since different people applying the test will obtain different values because of a random process. One sub-division may lead to significant results where another does not. To overcome the problem of uniqueness of the result, this study proposed to modify the random selection of the observation for the subsamples based on the ranking procedure to lead for a unique result for each solution. The original and modified Moses test were tested on the same data set. The finding shows that the result for both tests is similar in terms of decision and conclusion. The analysis revealed that the modified Moses test based on ranking approach has a smaller sum of squared values compared to the original Moses test. Thus, the variability of data for each subsample is decreased as well. Ranking approach can be used as an alternative to replacing the random procedure of selecting observations for subsample to overcome the problem of uniqueness in the test statistic.

Keywords: Equality of dispersion parameters, Moses test, random procedure, Sum of squared

1. Introduction

The Moses test is known as a nonparametric method for dispersion, completely free from the location parameter of the distribution. The Moses test for equal variability is a variant of tests of dispersion that have been developed for contrasting the variances of two independent samples. Moses test was proposed in 1963 to test the equality of two dispersion parameters (Moses, 1963). This test has a similar function with the Ansari-Bradley test, which formerly proposed by Ansari and Bradley in 1960 (Ansari & Bradley, 1060). The difference between Moses and Ansari-Bradley tests is that the Moses test assumes that the two unknown populations have unequal medians while Ansari-Bradley test assumes the medians are equal. To apply the Moses test, there are few assumptions to fulfill which are the data have two random samples from populations 1 and 2, the distribution of the populations must be continuous and have the same shape, and the two samples are independent of each other (Daniel, 2016). An advantage of this test is that it does not assume the equality of the location parameters. However, this method has several disadvantages concerning the uniqueness of the result and applicability to small samples (Ushizawa & Sato, 2005). This method has different values of the statistical test since the subsample comes from the random procedure.

Moses test for equal variability does not rank the original interval or ratio scores instead ranks the sums of squared difference or deviation scores. For this reason, some researchers' such as Siegel and Castellan (1988) and Grawe (2016) categorize the Moses test for equal variability as a test of interval or ratio data. However, Sheskin (2011) categorized the Moses test for equal variability as a test of ordinal data, since the ranking procedure is a fundamental component of the test protocol.

Lehmann (1951) suggest to apply the Moses test if the samples are taken from symmetric distributions. Emerson and Moses (1985) claim that Mann Whitney test is more powerful than other methods involving the comparison of two or more ordered categories. Based on the research conducted by Olejnik and Algina (1987) with respect to power estimates of selected parametric and nonparametric tests of scale, the development of rank tests of scale has been established from the sensitivity of non-normality of several variance tests. Moses (1963) alerted that since it might be influenced by differences in group location parameters and group variances, therefore, the procedure would be of limited value. Apart from that, if there exists non-overlapping between the distribution of two populations and the sample sizes were equivalent, the degree of dispersion within each group does not affect the order of observations and the test statistics.

The Moses test has been used in principal component scores (PC-scores) to check the intermediate latent root equality of two covariance matrices assuming non-normal distribution by using three different tests. The Moses test is free from position parameters for PC-scores distribution, but the researchers suggested that the subgroup size should be large but must not be greater than 10 (Hollander, 1968). The comparison between Moses test with Ansari-Bradley and F tests are made in term of efficiency (Ushizawa & Sato, 1998). The Monte Carlo simulation has been used to compare the reliability of the testing comparison between these three tests. Their study concludes that when there is a departure from the normal distribution, the sensitivity of Moses test will not exist. Moses test is useful when the principal component distribution kurtosis is greater than the normal distribution. Ushizawa and Sato (2005) modified the Moses test to solve the problem of uniqueness of the result. The new procedure constructs subgroups for the test, based on random resampling with replacement, not on dividing observations. Simulation results revealed that the modified method has good qualities compared with F-test, Ansari-Bradley test, or the bootstrap method.

Moses test has wider applicability since it does not need to fulfill the assumption of known or same location parameter to test the equality of the dispersion parameter. Moses test is relatively easy to determine compared to a more effective alternative but the test statistics for each researcher varies and that means the significant result also may be varied due to the randomization process that is applied to the subpopulation (Daniel, 2016). The equality of position parameters and dispersion parameters are evaluated simultaneously. Hollander (1968) suggested that some test such as Mann Whitney and Moses tests could be used to test the equality of dispersion parameters. Daniel (2016) mentioned that these two tests are uncorrelated and asymptotically independent when the dispersion of parameters are equal, and the sample population is symmetric.

An obvious problem associated with the Moses test for equal variability is that its result is dependent on the configuration of the data in each of the random subsamples employed in the analysis. The computation of the test statistics involves the random selection of observations for the subsamples which is the number of subsamples obtained based on subsamples size. The subsamples size is flexible which means it will be decided by the researchers. Different researchers might choose different subsamples size. It is possible that an analysis with one set of subsamples may yield a different result and the contrary is true (Sheskin, 2011). The random procedure in selecting the observations for subsamples may lead to insignificant statistical results since different researchers use different values of test statistics. In other word, one subdivision may lead to significant results while another does not. Therefore, to solve the problem of uniqueness of the result, this study proposed to modify the random selection of observations for the subsamples to get consistent results for the test statistics based on the ranking procedure where observations from the two samples were ranked from the smallest to the largest without combining the 2 samples. Next, the observations of each sample are chosen based on that order to be fitted into each subsample. More explanation on this method are discuss in the next section.

This paper is organized as follows. Section 2 describes methodology which includes the original Moses Test, modified method, and secondary data. Results and discussion are presented in Section 3 followed by the conclusion in Section 4.

2. Methodology

Moses test is applicable for two independent samples to test the equality of dispersion parameters. This test has greater and wider applicability since it does not assume the equality of median between two populations, in a real-world application, the assumption regarding the median equality is usually violated. This section will explain the original Moses test and the modification methods.

2.1 Moses Test

The application of Moses test depends on the fulfillment of the following assumptions: 1) the data consist of two samples that are randomly selected namely $X_1, X_2, \dots, X_{n1}$ and $Y_1, Y_2, \dots, Y_{n2}$ that come from populations 1 and 2 respectively, 2) have a continuous population distribution and the same shape (symmetric), 3) measured on at least an interval scale, and 4) the two samples must be independent. The procedure to compute the test statistic for Moses test are as follows:

i. The observations of X and Y are divided randomly into m_1 and m_2 subsamples of k size. The k will be decided by the researcher while the number of subsamples for X and Y is obtained by dividing the number of samples with k.

ii. Leftover observations should be discarded. It is recommended that the k should be as large as possible, but not more than 10 so by that m_1 and m_2 are large enough to derive meaningful results (Shorack, 1969).

iii. After obtaining the number of subsamples, the observations in X and Y are randomly selected. The number of observations in each subsample is based on k.

iv. Next, compute the sum of squared deviations of observations from their mean in each subsample. In term of this, compute the numerator of the familiar sample variance; the numerator has the form $\sum(X-\bar{X})^2$ or $\sum(Y-\bar{Y})^2$. Designate the m_1 sums of squares obtained from the subsamples of X's by C_1, C_2… C_{m1}. Similarly, designate the m_2 sums of squares computed from the subsamples of Y's by D_1, D_2… D_{m2}.

v. The application of the Mann-Whitney test is used in this test by letting the C's and D's take the role of the X's and Y's respectively and letting n_1 and n_2 replaced by m_1 and m_2.

vi. Then the test statistic for the Moses test for equal variability is computed with the Mann–Whitney test as shown in Equation (1):

$$T = S - \frac{m_1(m_1+1)}{2} \tag{1}$$

Eq. (1) is applied to a data set to validate the Moses test, where S is equal to the sum of the ranks assigned to the sums of squares computed from X's subsamples.

2.2 Moses Test Based on Ranking

Firstly, before dividing the observation of X and Y into subsamples, the size of subsamples which is k needs to be considered to ensure the number of subsamples is sufficient. Sample size k needs to be as large as possible but not more than 10. By that, the leftover observations are discarded to maintain the completeness of sample data (Shorack, 1969).

Next, in the original procedure of the Moses test, the observations for each subsample are selected randomly. In this modification part, without combining the sample of X and Y, rank the observations of these two samples from the smallest observation to the largest. Hence, the observations of sample X and Y are chosen based on that order to be fitted into each subsample, m_1 and m_2. The mean and sums of squares from each subsample is obtained to compute the test statistic.

2.3 Secondary Data

Performance of the original Moses test and the modified method was measured and compared based on the secondary data obtained from McGuffin et al. (1974). The data is about systolic blood pressure that comes from cardiovascular findings in acromegaly patients. Acromegaly is one of the hormonal disorders caused by chronic, excessive growth hormone secretion through the pituitary gland. This disease is most common in the middle ages with the same level of males and females affected. In this study, two groups of subjects were involved which are normotensive and hypertensive patients. Normotensive patients are patients with normal blood pressure while hypertensive patients are patients that have high blood pressure. There are 10 patients who were normotensive and 13 patients that have hypertensive. The analysis then has been set with significance level as 0.05.

3. Results and Discussion

This section discussed the result of analysis and some discussion by using the original Moses test and the modified Moses test. Moses test is being conducted as follows:

Table 1. Two groups of patients with systolic blood pressure (mmHg)

Normotensive Patients (X)	122	110	140	130	140	110	120
	105	98	140				
Hypertensive Patients (Y)	160	140	150	140	150	160	220
	155	150	170	180	210	150	

The hypothesis statement for this problem is

$H_0: \sigma_1 \geq \sigma_2$

$H_1: \sigma_1 < \sigma_2$

According to the original Moses test, the test statistic was obtained by choosing the observations from each sample into subsample randomly as shown in Table 2 and Table 3.

Table 2. Sample X, k=3, m_1=3 (one observation is discarded)

Subsample	Observations	Sums of square (SS)
1	130, 98, 122	554.67
2	140, 110, 105	716.67
3	110, 120, 140	466.67

Table 3. Sample Y, k=3, m_2=4 (one observation is discarded)

Subsample	Observations	Sums of square (SS)
1	160, 150, 170	200
2	210, 150, 150	2400
3	140, 220, 155	3616.67
4	150, 140, 180	866.67

Arrange the sum of squares for both samples in ascending order and compute the test statistic as shown in Table 4.

Table 4. Sums of squares in order of rank and test statistics value

Sums of squares of X	**Rank**	**Sums of squares of Y**	**Rank**
554.67	3	200	1
716.67	4	2400	6
466.67	2	3616.67	7
		866.67	5
Total (S)	9		
Test statistic (T)	3		
p-value	More than 0.1		

From Table 4, the p-value is more than 0.1 and greater than the significance value. Therefore, we fail to reject the null hypothesis. This study concludes that there is not enough evidence to indicate that hypertensive patients have a greater dispersion of systolic blood pressure than normotensive patients.

Based on the Modified Moses test, rank the observations in ascending order as shown in Table 5. Divide the observations for each sample into subsample based on its order from smallest to largest as presented in Table 6 and Table 7. Then calculate the test statistic as given in Table 8.

Table 5. Two groups of patients with systolic blood pressure (mmHg) in ascending order

Normotensive Patients (X)	98	105	110	110	120	122	130
	140	140	140				
Hypertensive Patients (Y)	140	140	150	150	150	150	155
	160	160	170	180	210	220	

Table 6. Sample X, k=3, m_1=3 (one observation is discarded)

Subsample	**Observations**	**Sums of square (SS)**
1	98, 105, 110	75.69
2	110, 120, 122	82.67
3	130, 140, 140	66.67

Table 7. Sample Y, k=3, m_2=4 (one observation is discarded)

Subsample	**Observations**	**Sums of square (SS)**
1	140, 140, 150	66.67
2	150, 150, 150	0
3	155, 160, 160	16.67
4	170, 180, 210	866.67

Table 8. Sums of squares in order of rank and test statistics value

Sums of squares of *X*	**Rank**	**Sums of squares of *Y***	**Rank**
66.67	3.5	0	1
75.69	5	16.67	2
82.67	6	66.67	3.5
		866.67	7
Total (S)	14.5		
Test statistic (T)	8.5		
p-value	More than 0.1		

Table 8 presents the p-value is more than 0.1, where obviously we fail to reject the null hypothesis. Therefore, there is not enough evidence to indicate that hypertensive patients have a greater dispersion of systolic blood pressure than normotensive patients.

The findings show that original and modified Moses test give a similar decision and conclusion in the hypothesis testing procedure. However, the original Moses test provides inconsistent result for the test statistics. The sum of the square value for each subsample is high since the mean value can be greatly influenced when the observations are chosen randomly. The test statistic value varies based on random selection for subsamples which may lead to inconsistent statistical result. On the other hand, the modified Moses test with ranking approach produced smaller sum of square and provide a consistent result for the test statistics. The value of the sum of squares is reduced, hence the variability of data for each subsample is decreased as well.

4. Conclusion

In this study, the random procedure in selecting observations for each subsample was replaced with the ranking procedure in which the observations in the subsample are selected by arranging the observation from smallest to largest. The result based on the modified Moses test with ranking approach become more standardized and consistent. By applying this modified method, we obtain similar value for the test statistic and the same conclusion. It can be observed that the value of the sum of the square in the modified method based on ranking is reduced. As the sum of square value becomes smaller, it implies that the method achieves less variability based on the observations in the subsample. Therefore, this modification can be used as an alternative to replacing the random procedure of selecting observations for subsample to overcome the problem of uniqueness in the test statistic.

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