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Application of Asymmetric GARCH Mixture Models to Export Price of Natural Gas in Nigeria

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RESEARCH ARTICLE

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Abstract

This study aims to examine the volatility of Nigeria's natural gas export prices from 2015 to 2022 using asymmetric GARCH mixture models. It highlights significant price fluctuations, with a pronounced upward trend starting in 2020, likely driven by global market shifts such as the Russia-Ukraine conflict and related energy supply disruptions. The tGARCH-eGARCH model outperformed other GARCH mixtures, yielding the lowest Deviance Information Criterion (DIC) value of -949.3355. This suggests it best captures the volatility characteristics by effectively modeling heavy-tailed distributions and asymmetries in price behavior. The study reveals that Nigeria's natural gas prices respond asymmetrically to market shocks, with negative events (like geopolitical tensions and supply disruptions) causing more significant volatility spikes than positive ones. High persistence in volatility, particularly evident in the eGARCH-eGARCH model, indicates that once volatility rises, it remains elevated for extended periods. The ARCH LM test detected significant effects at lag orders 4 and 8, but these effects dissipated by lag 12, indicating that recent shocks have a strong but diminishing impact on volatility over time. The data showed a slight right skew (skewness = 0.753), suggesting that extreme price increases were more frequent than sharp declines, possibly due to market asymmetries and supply constraints. The study concludes that the persistent and asymmetric nature of Nigeria's natural gas price volatility mirrors broader global trends influenced by geopolitical events and macroeconomic instability. The findings underscore the importance of using flexible, hybrid models like tGARCH-eGARCH for accurate volatility forecasting and the study provides valuable insights for policymakers, investors, as well as risk managers navigating the complexities of the natural gas market.

Keywords: Criterion, Shocks, Volatility, Deviance, Leverage, Energy

1. Introduction

The volatility of natural gas export prices is a critical concern for Nigeria, given its substantial reliance on natural gas revenues. Understanding and accurately modeling this volatility is essential for policymakers and stakeholders to make informed decisions. Recent studies have emphasized the effectiveness of Generalized Autoregressive Conditional Heteroskedasticity (GARCH) models in capturing the volatility dynamics of energy commodities. Victor-Edema and Wariboko (2023) investigated the performance of symmetric and asymmetric GARCH models in Nigeria's crude oil markets. Their findings indicated that the Threshold GARCH (TGARCH) model, under a student's-t distribution with a fixed parameter degree of freedom, was particularly adept at modeling volatility in crude oil prices. This suggests that asymmetric GARCH models, which account for the possibility that positive and negative shocks have different effects on volatility, may also be suitable for analyzing natural gas price volatility (Raheem *et al*, 2023). Furthermore, the global natural gas market has experienced significant fluctuations in recent years. According to the International Energy Agency's Gas Market Report for the first quarter of 2024, price volatility moderated from an all-time high of 160% in 2022 to an average of 75%, remaining well above the 35% average observed during 2016-2020 (Guo 2023). These heightened levels of volatility underscore the importance of employing robust models to understand and predict price movements. In the context of Nigeria, the natural gas sector plays a pivotal role in the nation's economy. A report by PwC highlighted that Nigeria ranked fourth globally in natural gas exports and controlled about 7% of the total global export of natural gas as of 2017. This significant position in the global market makes it imperative to analyze and forecast natural gas price volatility accurately (Raheem *et al*, 2023).

Volatility is a measure of dispersion around the mean or average return (Eriyeva and Okoli, 2021). It can be measured using the standard deviation, which signals how tightly the price of a stock is grouped around the mean or moving average (MA). When prices are tightly bunched together, the standard deviation is small. When prices are widely spread apart, the standard deviation is large. Volatility possesses a number of stylized facts which make it inherently more forecastable. Predicting volatility is quintessential to effective and efficient allocation of risk and optimal participation in the financial market (Eriyeva and Okoli, 2021, Kennedy *et al,* 2023). The more volatile an asset is, the more the returns potential. This research thus investigates the volatility of natural gas export prices in Nigeria. The study used comparative volatility capturing models which have been established to yield better result in the modeling of volatility clustering and time varying volatility. The research result and framework would be useful to the government to ascertain expected returns as a function of the volatility of the asset. Consequently, the government can effectively plan its economy based on these projected returns. Given these considerations, this study aims to evaluate Nigeria's natural gas export price data using asymmetric GARCH mixture models. The study seeks to provide a more nuanced understanding of the volatility patterns inherent in the data, thereby offering valuable insights for policymakers and investors navigating the complexities of the natural gas market.

1. Empirical Review of GARCH Family Models

The GARCH (Generalized Autoregressive Conditional Heteroskedasticity) model, developed by Otto and Schmid (2023) extends the ARCH model by modeling conditional variance as a function of past squared errors and past variances. The conditional variance, σ_t^2, is influenced by past innovations and model parameters, making the process dependent and non-identically distributed. Rahman et al (2023) generalizes the pure ARCH model into the GARCH framework, analogous to the transition from MA to ARMA models. The GARCH(1,1) model is expressed as:

$$\sigma_t^2 = \omega + \alpha_t \varepsilon_{(t-1)^2} + \beta_t \sigma_{(t-1)^2} \qquad (1)$$

Other symmetric models include the Integrated GARCH (IGARCH) model, characterized by persistent variance where shocks to volatility may have long-lasting effects (Otto and Schmid, 2023). The IGARCH(1,1) model is given by:

$$\sigma_t^2 = \alpha_t \varepsilon_{(t-1)^2} + \beta_t \sigma_{(t-1)^2} \tag{2}$$

When ω = 0, the process is a martingale, and while it exhibits stationarity and ergodicity, it does not behave like a random walk.

To address the symmetric response limitation of GARCH models, Otto and Schmid (2023) introduced the Exponential GARCH (EGARCH) model. The EGARCH(1,1) model captures asymmetries in volatility by modeling the logarithm of the conditional variance:

$$\log\left(\sigma_t^2\right) = \omega + \beta \log\left(\sigma_{t-1}^2\right) + \gamma\left|\varepsilon_{t-1}\right| + \theta\varepsilon_{t-1} \tag{3}$$

where:

σ_t^2 is the conditional variance of the innovation at time t.

ω is the constant that determines the long-run average level of log-volatility.

$\beta \log\left(\sigma_{t-1}^2\right)$ is the persistence term that captures volatility persistence over time.

$\left|\varepsilon_{t-1}\right|$ The lagged innovation (shock) from the mean equation which represents unexpected movement in the series at time t-1.

$\gamma\left|\varepsilon_{t-1}\right|$ is the magnitude effect that measures the impact of the size of past shocks, regardless of sign.

$\theta\varepsilon_{t-1}$ is the asymmetry/leverage term that captures asymmetric volatility response to shocks.

This structure allows for asymmetric responses to positive and negative shocks without requiring parameter restrictions to ensure non-negativity (Kennedy *et al*, 2023).

The Mixed Normal GARCH (MixN-GARCH) model assumes the conditional density of returns and follows a finite normal mixture distribution as well as enhancing flexibility in capturing volatility dynamics thus;

$$f(t) = \sum_{i=1}^{n+1} \pi_{ti} \Phi\left(\mu_{ti}, \sigma_{ti}^2\right) \tag{4}$$

where:

$f(t)$ is the probability density (or mass) function of the time series observation at time t and π_{ti} are mixing weights (or regime probability) of component i at time t.

μ_{ti} is the mean of the time series in regime i at time t.

σ_{ti}^2 is the variance capturing volatility or noise in that regime and σ_{ti}^2 follows a GARCH (p, q) process.

The evolution of symmetric GARCH models reflects the ongoing effort to accurately capture and forecast volatility in financial time series. Beginning with the foundational GARCH (1,1) model, which builds on the ARCH framework by incorporating both lagged squared residuals and past variances, the family has expanded to include more nuanced approaches like IGARCH and MixN-GARCH (Onyeka-Ubaka,. and Anene, 2020; Otto and Schmid, 2023, Kennedy *et al*, 2023). These models accommodate features such as long memory in volatility and more flexible error distributions. While the standard GARCH and IGARCH models assume symmetric responses to shocks, the development of models like EGARCH has introduced mechanisms to address this limitation by allowing for asymmetric volatility behavior.

Additionally, the incorporation of mixture distributions in MixN-GARCH enhances the model's ability to reflect empirical return characteristics such as leptokurtosis and volatility clustering. Overall, symmetric GARCH models, including their advanced variants, remain essential tools in econometrics, offering a robust foundation for modeling financial volatility while continuously evolving to better capture real-world complexities.

Despite Nigeria's increasing reliance on natural gas exports, empirical evidence on the volatility dynamics of its natural gas export prices remains limited. Existing studies largely adopt symmetric GARCH models, which assume uniform volatility responses to price shocks, an assumption that may be restrictive for natural gas markets exhibiting asymmetry and persistence. Although asymmetric GARCH models relax this constraint, they are typically estimated within a single regime framework and may fail to capture regime dependent volatility behavior. Moreover, the application of mixture GARCH models to Nigeria's natural gas export prices is scarce. This study addresses this gap by comparatively evaluating symmetric, asymmetric, and mixture GARCH specifications to better characterize volatility clustering, asymmetry, and regime shifts.

2. Materials and Methods

Nigeria is located in West Africa, along the Gulf of Guinea on the Atlantic Ocean. Geographically, it lies between latitudes 4° and 14° North and longitudes 3° and 15° East. It shares borders with Niger Republic to the north, Chad and Cameroon to the east, Benin Republic to the west, and is bounded to the south by the Atlantic Ocean. Nigeria is a federal republic, comprising 36 states and the Federal Capital Territory (FCT), Abuja. It is the most populous country in Africa and is often referred to as the "Giant of Africa" due to its size, population, and economic potential. The country is marked by significant geopolitical, ethnic, religious, and cultural diversity, making it one of the most complex and influential nations on the continent. Nigeria is home to over 250 ethnic groups, with the three major ones being the Hausa-Fulani in the North, Yoruba in the Southwest, and Igbo in the Southeast. Religiously, the population is primarily divided between Islam and Christianity, with a smaller percentage adhering to African Traditional Religions (ATR).

Despite being richly endowed with natural energy resources, notably oil and gas, Nigeria faces significant infrastructural challenges in the energy sector. A large portion of the population still lacks access to modern energy services, relying heavily on traditional biomass fuels and privately generated power. Nevertheless, Nigeria holds great potential in renewable energy, particularly solar power, which remains underutilized.

Data for this project was sourced from Investing.com (InvestingNG.com Stock Market Quotes & Financial News), which provides a wide range of datasets useful for research purposes. The obtained data were subjected to descriptive statistics, test of stationary and Unit root test. Also adopted were Augmented Dickey Fuller test and Deviance Information Criteria (DIC) which was used to select the best GARCH mixture model.

2.1 Stationarity and Unit Root Testing

A time series is said to be stationary if its mean, variance, and autocovariances are constant over time. Most standard time series methods, including GARCH modeling, require stationarity to ensure valid inference. Non-stationarity often arises from deterministic trends or stochastic trends (unit roots). To address this, we first transform the natural gas price series into log-returns and then formally test for stationarity.

The Augmented Dickey–Fuller (ADF) test (Dickey & Fuller, 1979) is employed to detect a unit root. The test augments the basic Dickey–Fuller regression with lagged differences to account for serial correlation which can be written as:

$$\Delta y_t = \mu + \gamma t + \delta y_{t-1} + \sum_{i=1}^{p} \phi_i \Delta y_{t-1} + \mu_t \tag{5}$$

where:

$\Delta y_t = y_t - y_{t-1}$ is the first difference of the series,

μ is a constant (drift term),

γt represents a deterministic time trend (if included),

δ is the coefficient on the lagged level of the series, used to test for a unit root,

ϕ_i are coefficients of lagged differences to account for serial correlation, up to lag p,

μ_t is a white noise error term.

Each version of the test has its own critical value which depends on the size of the sample. In each case, the null hypothesis is that there is a unit root, $\delta = 0$.

2.2 Asymmetric GARCH Specifications

2.3.1 Exponential GARCH (EGARCH)

The **EGARCH** model of Nelson (1991) specifies the logarithm of conditional variance, ensuring positivity without imposing parameter restrictions, Boubaker & Makram (2024):

$$\log\left(\sigma_t^2\right) = \omega + \beta \log\left(\sigma_{t-1}^2\right) + \gamma \left|z_{t-1}\right| + \theta z_{t-1} \tag{6}$$

where

$$z_{t-1} = \varepsilon_{t-1} / \sigma_{t-1} \tag{7}$$

The parameter θ captures leverage effects, while β measures volatility persistence. The GJR-GARCH model (Glosten et al., 1993) captures asymmetry through an indicator function:

$$\sigma_t = \omega + \alpha \left|\varepsilon_{t-1}\right| + \gamma \left|\varepsilon_{t-1}\right| \mathrm{I}\left(\varepsilon_{t-1} < 0\right) + \beta \sigma_{t-1} \tag{8}$$

This formulation allows volatility to respond differently depending on the sign of past innovations.

2.3.2 GJR GARCH

The GJR model of Glosten, Jagannathan, and Runkle (1993) is also able to capture the asymmetry in the conditional volatility process. This model is given by:

$$h_{k,t} = \alpha_{0,k} + \left(\alpha_{1,k} + \alpha_{2,k} \left\|\left\{y_{t-1} < 0\right\}\right)y_{t-1}^2 + \beta_k h_{k,t-1}, fork = 1 \tag{9}$$

where | $\{\cdot\}$ is the indicator function taking value of one if the condition holds, and zero otherwise and $\theta_k = \left(\alpha_{0,k}, \alpha_{1,k}, \alpha_{2,k}, \beta_k\right)^T$.

The parameter α controls the degree of asymmetry in the conditional volatility response to the past shock in regime k. To ensure positivity, we require that:

$$\alpha_{0,k} > 0,\ \alpha_{1,k} > 0,\ \alpha_{2,k} \geq 0,\ \beta_k \geq 0$$

Covariance-stationarity in each regime is obtained by requiring that:

$$\alpha_{1,k} + \alpha_{2,k}\mathrm{E}\eta_{k,t}^2 \left\|\left\{\eta_{k,t}k, t < 0\right\} + \beta_k < 1\right.$$

2.3.3 TGARCH

Threshold GARCH model (TGARCH) was introduced in (1994) by Zokian. TGARCH specifies that the conditional volatility is the dependent variable instead of the conditional variance. This model is given by:

$$h_{k,t}^{\frac{1}{2}} = \alpha_{0,k} + \left(\alpha_{1,k}\mathrm{I}\left\{y_{t-1} \geq 0\right\} - \alpha_{2,k}\mathrm{I}\left\{y_{t-1} < 0\right\}\right)y_{t-1} + \beta_k h_{k,t}^{\frac{1}{2}}, fork = 1,\ldots,k \tag{10}$$

where:

$h_{k,t}^{\frac{1}{2}}$ is the conditional standard deviation of series k at time t

y_{t-1} is the observed value of a financial time series (market return or asset return) at time *t-1*

$\mathrm{I}\left\{y_{t-1} \geq 0\right\}$ is the indicator function equal to 1 if $y_{t-1} \geq 0$, and 0 otherwise

$\mathrm{I}\left\{y_{t-1} < 0\right\}$ is the indicator function equal to 1 $y_{t-1} < 0$, and 0 otherwise

$\alpha_{0,k}$ is the constant term for series k (related to the long-run level of volatility)

$\alpha_{1,k}$ is the reaction coefficient to positive shocks $y_{t-1} \geq 0$ in series k

$\alpha_{2,k}$ is the reaction coefficient to negative shocks $y_{t-1} < 0$ in series k

β_k is the persistence parameter (autoregressive coefficient on lagged volatility) for series k. k is the total number of equations (assets or volatility components)

To ensure positivity, we require that

$$\alpha_{0,k} > 0,\ \alpha_{1,k} > 0,\ \alpha_{2,k} > 0,\ \beta_k \geq 0\,.$$

2.3.4 Mixture GARCH Models

To account for regime heterogeneity, mixtures of asymmetric GARCH models were estimated. The conditional density of the innovation is specified as a finite mixture:

$$f\left(\varepsilon_t\right) = \sum_{i=1}^{K}\pi_i \mathrm{N}\left(0, \sigma_{i,t}^2\right),\ \sum_{i=1}^{K}\pi_i = 1 \tag{11}$$

where πi represents the mixing probability of regime i, and each regime follows an asymmetric GARCH process.

The following mixture specifications were considered:

EGARCH–EGARCH
GJR-GARCH–GJR-GARCH
TGARCH–TGARCH

2.3 Parameter Estimation

Model parameters were estimated using Bayesian and Quasi-Maximum Likelihood Estimation (QMLE) under conditional normality. QMLE provides consistent estimates even when the true innovation distribution departs from normality, making it suitable for exchange rate data. The Bayesian estimation was implemented using the MSGARCH package of the R programming software. The prior distribution was assumed normal for the duo mixtures. 500 large samples of our data set were simulated using the Markov Chain Monte Carlo (MCMC) simulation. The control parameters were varied until a suitable specification was created. The models were fitted on the MCMC simulated data.

2.4 Model Selection and Evaluation

Model performance was evaluated using the Deviance Information Criterion (DIC), which balances model fit and complexity. DIC is particularly appropriate for mixture and hierarchical models. The model with the lowest DIC value was selected as the preferred specification.

$$D(\theta) = -2\log(p(y\,|\,\theta)) + C \tag{12}$$

where:
y are the data, θ are the unknown parameters of the model, $p(y\,|\,\theta)$ is the likelihood function and C is a constant.

Data obtained were subjected to descriptive statistics and time series plots using MINITAB while the mixture GARCH model using different combinations (eGARCH- gjrGARCH, tGARCH- eGARCH, tGARCH- gjrGARCH, tGARCH- tGARCH and eGARCH- eGARCH) and Gaussian errors were obtained using R programming software, JMulTi Econometric Software (version 4.24). The Deviance Information Criterion (DIC) for each of the combinations were obtained via the diagnosis analysis.

3. Results and Discussion

The smallest value ($1.495) in the dataset is the lower bound of the range and provides a sense of the lowest observed outcome. The median ($2.739) represents the midpoint of the data when sorted, indicating that 50% of the values are below $2.739 and 50% are above. The proximity of the median to the mean (2.725) suggests a somewhat symmetric distribution, although the skewness metric provides more insight into this. The mean, $2.725, the arithmetic average of the dataset is very close to the median (Table 1). This implies that extreme values (outliers) are not heavily distorting the central tendency, but the skewness metric suggests a slight right-skew.

Table 1. Descriptive Statistics of Natural Gas Export Price (4/01/2015- 19/09/2021)

Minimum	Median	Mean	Standard Deviation	Skewness	Maximum
1.495	2.739	2.725	0.575	0.753	5.105

The variance of the data, 0.331 measures the variability of the data points and it suggests a very low variability since the value is close to zero. The mean price of gas established in the current study is lesser than the average price of gas ($2.839) reported for the region (NBS, 2023). The disparity in the gas prices in the 2 studies may be attributable to the sources of the data as well as the effects of fuel subsidy removal in Nigeria (Evans *et al*, 2023). A positive skewness of 0.753 obtained for this study indicated a slight right-skewed data (Table 1). This portends that the dataset has a longer tail on the higher end and could be influenced by the presence of outliers or a natural asymmetry in the data. The time series plot of the gas export price of natural gas from 2015 to 2022 showed that the gas price was noisy (that is exhibits fluctuations over time) with clear peaks and troughs (Figure 1). There was also a notable upward trend starting around 2020 reaching its maximum in 2022. Varying levels of volatility was visible with some periods exhibiting sharp increases and decreases (in 2017 and 2019). The period between 2020 and 2022 appears to show increased upward movement, possibly indicative of heightened activity or changes in the underlying process. This heightened volatility mirrors broader global trends, particularly those stemming from the Russia-Ukraine conflict and the ensuing disruptions in energy supply chains. According to the International Energy Agency (IEA, 2022), global gas markets witnessed unprecedented price swings due to restructured supply networks and increased European demand to offset reduced Russian gas imports. It was also visible that the data seems to exhibit cyclical behavior with recurring increases and decreases over time. However, the cycles are not perfectly regular. The highest point occurs near the end of the time frame (2022), aligning with the maximum value of 5.105 in the descriptive statistics. The lowest periods occur around 2016 and early 2020, where the value approaches the minimum of 1.495. The cyclical behavior of this time plot conforms to Raheem *et al* (2023) where the price of oil and gas for some companies returned a noisy trend. This can be hinged on fluctuation in the inflation rate in the country (Evans *et al*, 2023). The cyclic trend obtained in the present study however conflict with continuous trends obtained in Victor-Edema and Wariboko (2023). This disparity might be hinged on difference in the market behaviours of the two commodities under study. Our present research focuses on gas while Victor-Edema and Wariboko (2023) concentrated on fuel price

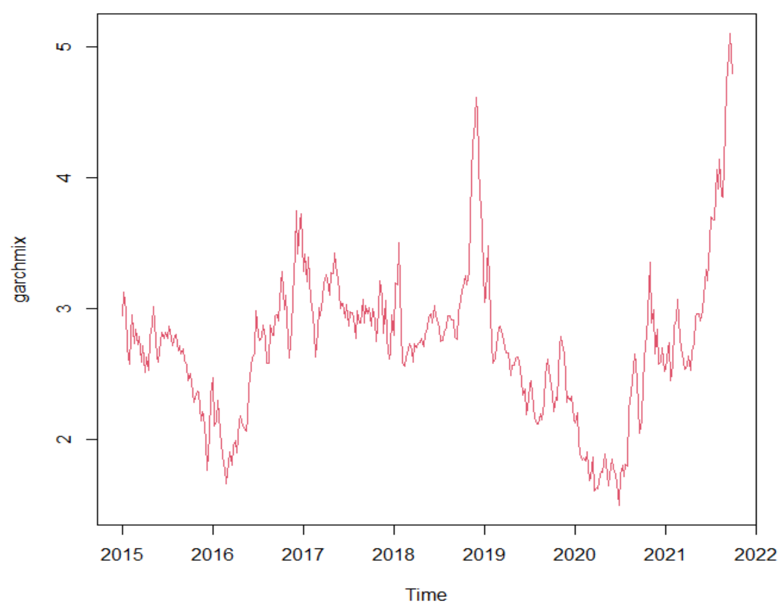

Figure 1. Time plot of Export Price for Natural Gas in Nigeria

Mean gas price fluctuate across months and weeks showing variations in the dataset while the standard error (SE) values indicate the precision of these means. The lower SE implies more confidence in the estimate. Higher variance for month ($M9 = 0.573$) suggest greater spread in the data and the variance is relatively low in some other month (like $M10 = 0.128$) indicating closely clustered data points. Negative skewness for month ($M2 = -0.746$) means the distribution has a longer tail on the left while positive skewness ($M9 = 2.05 -$ Table 2) indicates a longer tail on the right and this suggests outliers or higher values are more frequent. The shift from negative skewness in the months (M1–M6) to positive in the months (M7–M12) might indicate seasonal trends. Weekly data show relatively stable means (~2.7) but some fluctuations in skewness. Higher skewness in week five ($W5 = 1.399$) suggests a possible outlier or unusual data distribution (Table 2). Recent studies highlight how seasonality affects data distribution in various fields, including economics, climate studies, and health statistics (Smith *et al.,* 2021, Zhang and Liu, 2023). Monthly variations often follow cyclical trends, with skewness shifts signaling changes in external factors (Jones *et al.*, 2022). Fluctuations in variance indicate potential external influences such as economic cycles or environmental factors (Lee *et al.*, 2021). Positive skewness often correlates with unexpected peaks in datasets, requiring advanced modeling techniques like generalized linear models.

Descriptive statistics revealed a slight right skew in the price data (skewness $= 0.753$), suggesting that extreme price increases occurred more frequently than sharp declines. This skewness points to the presence of outlier events or market asymmetries. The right-skewed distribution of natural gas prices aligns with recent literature emphasizing asymmetry in energy price movements. Olanrewaju and Oseni (2022) observed that energy markets frequently exhibit right-skewness due to price spikes during periods of supply constraints or demand surges. This asymmetry underscores the need for models like tGARCH that can effectively capture tail behaviors.

Table 2. Descriptive Statistics of Natural Gas Export Price by Months (M) and Weeks (W)

	Months	N	Mean ± SE	Variance	Skewness
Months	M1	31	2.787 ± 0.090	0.245	-0.415
	M2	28	2.500 ± 0.082	0.188	-0.746
	M3	31	2.440 ± 0.090	0.253	-0.574
	M4	30	2.537 ± 0.081	0.195	-0.377
	M5	31	2.625 ± 0.087	0.252	-0.545
	M6	30	2.692 ± 0.103	0.299	-0.648
	M7	31	2.736 ± 0.100	0.32	0.302
	M8	31	2.904 ± 0.115	0.412	1.414
	M9	30	2.917 ± 0.141	0.573	2.05
	M10	31	2.833 ± 0.070	0.128	-0.351
	M11	30	2.951± 0.129	0.431	1.185
	M12	31	2.835 ± 0.133	0.479	0.644
Weeks	W1	81	2.734 ± 0.064	0.331	0.958
	W2	81	2.740 ± 0.066	0.351	0.737
	W3	81	2.713 ± 0.066	0.35	0.657
	W4	80	2.730 ± 0.062	0.312	0.539
	W5	28	2.676 ± 0.106	0.314	1.399

Low standard error obtained for some of the mean gas price in the present study suggests robust estimates, making data interpretation reliable (Rodríguez *et al.*, 2024). However, outliers in skewed distributions can distort means, necessitating non-parametric tests for better representation (Gomez and Patel, 2023).

3.1 Unit root test and Volatility Assessment

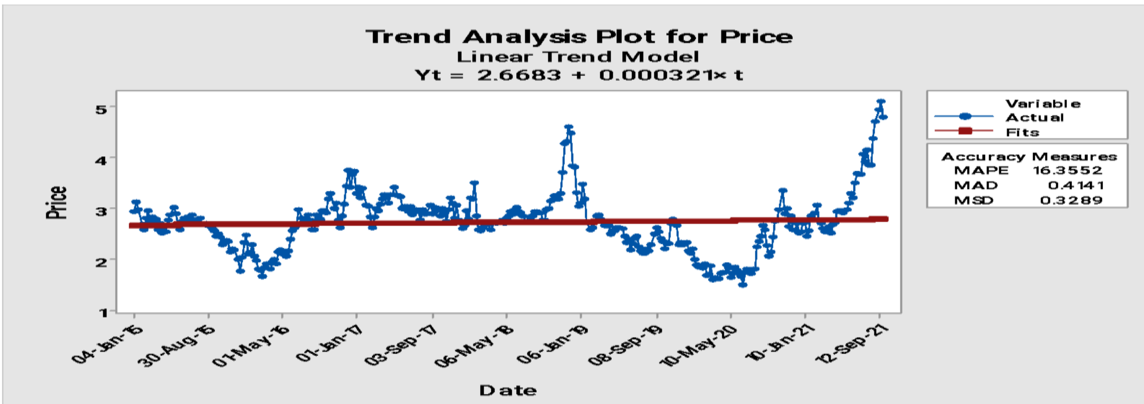

Figure 2: Trend Analysis of Export Price for Natural Gas in Nigeria

The time plot shows that the data is not stationary which implies that a stationarity test is to be carried out. To achieve this we carry out an Augmented Dickey Fuller test which is a test for stationarity.

H_0: No stationarity in the dataset

Vs

H_1: There is stationarity in the dataset

At $a = 0.05$, a p-value of 0.7034 indicates failure to reject the unit root null hypothesis. Consequently, the log-differenced series is used to achieve stationarity. Then, we carry out again the augmented Dickey Fuller test to see if our transformed data is stationary. At $a = 0.05$, the p-value for the test is 0.01. This implies that the data is stationary. We also move a step further to see if our data is suitable for a GARCH model, which means that the data must violate one of the assumptions of the regression model which is homoscedacity

To estimate the p-values for the ARCH LM (Lagrange Multiplier) test results, we use the chi-squared distribution. The LM test statistic follows a chi-squared distribution with degrees of freedom equal to the number of lags (order). The p-values for each order are as contained in Table 3. At lag order 4, **t**he very small p-value (< 0.001) indicates strong evidence of ARCH effects at this lag order. Similarly, the p-value is extremely small, showing significant ARCH effects for 8 lags order.

Table 3. ARCH LM TEST Effects at various Lags

Order	LM Statistic	P-Value
4	102.3	0.000 (significant)
8	43.1	0.000000841 (significant)
12	12.26	0.425 (not significant)

The p-value of 0.425 is much larger than the standard significance threshold (e.g., 0.05), suggesting that ARCH effects are not significant at this (12) lag order (Table 3). Based on these results, ARCH effects are highly significant for lag orders 4 and 8 but dissipate by lag 12. It is thus worthy to consider starting with a GARCH model using lower lag orders (e.g., GARCH(4, q)) for a better model fit. The pronounced ARCH effects at shorter lags suggest that recent price shocks exert a considerable influence on current volatility, a finding consistent with Raheem *et al.* (2023), who observed similar dynamics in African commodity markets. It was noted that short-term disruptions, such as sudden demand shifts or policy changes, disproportionately affect volatility, although their influence wanes in the longer term.

3.2 Parameter Estimates for Different GARCH Mixture Models

The parameter estimates for different GARCH mixture models applied to weekly natural gas price data spanning January 4, 2015, to September 19, 2021 are as provided (Table 4). It includes results for combinations of models such as eGARCH, gjrGARCH, and tGARCH. Five combinations of these GARCH models (eGARCH, gjrGARCH; tGARCH, eGARCH; tGARCH, gjrGARCH; tGARCH, tGARCH and eGARCH, eGARCH were adopted. The parameters represent various model characteristics, such as volatility persistence, asymmetry, and leverage effects. Deviance Information Criterion (DIC) values were used to assess model fit with lower values indicating better model performance among the models. The parameters generally vary significantly across models, reflecting their unique abilities to capture different aspects of the data. High persistence (values close to 1) is evident in combinations like eGARCH, eGARCH for some parameters. The results indicated that tGARCH and eGARCH combination has the lowest DIC (-949.3355), suggesting that it is the most parsimonious. This study uncovered that natural gas prices react asymmetrically to market shocks. Specifically, negative shocks such as geopolitical unrest or supply chain disruptions trigger greater volatility increases than positive events. This finding aligns with Li and Luo (2022), who demonstrated that adverse events, including supply shortages and political instability, exert a more pronounced influence on volatility within the global LNG markets compared to positive developments. This corroborates the study's conclusion that negative shocks significantly amplify volatility in Nigeria's natural gas exports.

Table Parameter Estimates of the GARCH Mixture Models for Weekly Data of Price of Natural Gas from 4th January, 2015 to 19th September, 2021.

PARAMETERS	GARCH MIXTURES				
	(eGARCH-gjrGARCH)	**(tGARCH-eGARCH)**	**(tGARCH-gjrGARCH)**	**(tGARCH-tGARCH)**	**(eGARCH-eGARCH)**
a_{01}	-2.6410 (0.0636)	0.0097 (0.0002)	0.0196 (0.0005)	0.0130 (0.0002)	-0.7249 (0.0058)
a_{11}	0.2631 (0.0178)	0.1746 (0.0029)	0.0509 (0.0037)	0.2152 (0.0021)	0.4023 (0.0071)
a_{12}	0.3513 (0.0130)	0.1472 (0.0048)	0.2814 (0.0078)	0.1567 (0.0038)	-0.0367 (0.0062)
b_1	0.5069 (0.0120)	0.6447 (0.0040)	0.4366 (0.0067)	0.6181 (0.0050)	0.8908 (0.0007)
x_1				1.1326 (0.0220)	0.9289 (0.0026)
a_{02}	0.0004 (0.0000)	-0.7327 (0.0079)	0.0005 (0.0000)	0.0604 (0.0018)	-0.5512 (0.0060)
a_{12}	0.1771 (0.0018)	0.3430 (0.0034)	0.2162 (0.0026)	0.4329 (0.0074)	0.4006 (0.0089)
a_{22}	0.0002 (0.0000)	0.0638 (0.0024)	0.0130 (0.0007)	0.4372 (0.0106)	0.1434 (0.0060)
b_2	0.7279 (0.0022)	0.8674 (0.0015)	0.6887 (0.0029)	0.3982 (0.0069)	0.8876 (0.0007)
x_2				5.9504 (0.1494)	0.9765 (0.0027)
P_{11}	0.2105 (0.0038)	0.2130 (0.0029)	0.1280 (0.0043)	0.8469 (0.0092)	0.4736 (0.0100)
DIC	-945.7289	-949.3355	-944.9389	-939.2135	-947.5793
Acceptance Rate	**27.2%**	**28.3%**	**28.0%**	**27.3%**	**28.1%**

NB: The values in the brackets provide the essential "margin of error" for each parameter, which help to ascertain the reliability of the model's estimated coefficients and the overall soundness of the result.

The acceptance rates for all models ranged from 27.2% to 28.3%, indicating the efficiency of the sampling process in the Bayesian framework. This analysis potentially can guide decisions about model selection based on criteria like DIC and the findings highlight the differences in how these GARCH mixtures model volatility clustering and tail behaviors in natural gas prices can vary. The mean equation parameter (MEP) −2.6410-2.6410 (eGARCH, gjrGARCH) might represent the intercept term, adjusted by standard errors for precision. Negative values suggest average weekly returns could be below zero (implying deficit). The α\alpha (Shock Sensitivity or ARCH Effect) which indicates how recent shocks (unexpected price changes) influence current volatility. Higher values of α\alpha, 0.2631 obtained for eGARCH in (eGARCH, gjrGARCH)) suggest that recent price shocks have a strong impact on volatility. The β\beta (Volatility Persistence) which reflects how much of the past volatility carries over into the future. The values close to 1, which is 0.8908 obtained for eGARCH in (eGARCH, eGARCH)) indicate high persistence, meaning volatility changes are long-lasting. γ\gamma (Leverage Effect for Asymmetric Models) captures asymmetry in the impact of positive against negative shocks. Negative shocks often increase volatility more than positive shocks of the same magnitude. Parameter like −0.7249, -0.7249 for (eGARCH, gjrGARCH) suggests pronounced asymmetry in how past shocks impact volatility. In mix model like (tGARCH, tGARCH), certain parameters represent tail behavior or skewness to accommodate the heavy tails or sharp spikes in price returns.

The analysis also revealed high volatility persistence, particularly in the eGARCH-eGARCH model, where the persistence parameter (β) approaches 0.89. This suggests that once volatility surges, it remains elevated for an extended period. Such persistence is consistent with the findings of Kilian and Zhou (2022), who noted that in energy markets, volatility shocks often persist, especially when tied to structural factors like long-term supply agreements and regulatory shifts. This enduring volatility poses challenges for risk management among exporters and policymakers, emphasizing the need for robust forecasting models such as those within the GARCH family.

Among the GARCH mixture models analyzed, the tGARCH-eGARCH combination yielded the lowest Deviance Information Criterion (DIC) value (-949.3355), signifying the best fit for capturing volatility characteristics in the data. This model's superior performance reflects recent advancements in volatility modeling, where mixture models adeptly capture both heavy-tailed distributions and leverage effects. This agrees with Zhang *et al.* (2023) highlighted that integrating models like tGARCH (which accounts for fat tails) with eGARCH (which addresses asymmetry) significantly improves forecasting accuracy, especially under extreme price fluctuations in commodity markets.

Parameters such as 0.5069 (shock sensitivity for tGARCH in this model) may indicate the model's ability to account for large price jumps. Mix models combine features of different GARCH types to better capture complex volatility structures. From this result, tGARCH is tail-heavy distributions to model extreme events while eGARCH is exponential form for asymmetry and non-negativity. gjrGARCH focus on leverage effects (different reactions to good vs. bad news). Parameters like 0.7279 (volatility persistence in tGARCH for the (tGARCH, gjrGARCH) model) reflect the lasting effects of volatility in these dynamic mixtures.

3.3 Model Fit and Deviance Information Criterion (DIC)

While DIC measures the overall model fit, the individual parameters (and their standard errors) provide insight into the model's ability to capture key volatility features. Smaller standard errors (like 0.0007 for persistence in (eGARCH, eGARCH)) suggest higher confidence in parameter estimates.

From these results, it is expedient to note how shocks influence volatility, how persistent volatility is over time and whether asymmetry exists in the data. It can also be understood how well the model accounts for extreme events in price behavior. The best model among the GARCH mixtures depends on the criteria used for evaluation. Based on these criteria, (tGARCH, eGARCH) is the best model for this dataset due to its lowest DIC (D Information Criteria).

4. Conclusion

In conclusion, the study's findings align with recent advancements in volatility modeling and trends in global energy markets. The persistent and asymmetric volatility observed in Nigeria's natural gas prices reflects broader market forces shaped by geopolitical tensions, supply chain disruptions, and macroeconomic instability. The superior performance of the tGARCH-eGARCH model highlights the necessity of employing flexible, hybrid models to accurately capture the intricate volatility patterns inherent in commodity markets. Utilizing asymmetric GARCH mixture models offers a robust framework for analyzing the volatility dynamics of Nigeria's natural gas export prices. These methodological advancements, combined with the complex interplay of global and local economic factors, emphasize the relevance of this approach. The results carry important implications for policymakers, investors, and risk managers, providing valuable insights for navigating the highly volatile natural gas market landscape.

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