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Assessing Volatility Measurements for Enhanced Silver Price Forecasting via Geometric Brownian Motion

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Abstract

The fluctuations in Malaysia's commodity prices pose challenges for investors and policymakers. creating a need for reliable forecasting models. While Geometric Brownian Motion (GBM) has been widely used for modeling commodity prices, limited studies have examined its forecasting performance under different volatility specifications for Malaysian silver prices. Therefore, this study aims to compare the forecasting performance of GBM under simple volatility and log volatility specifications in predicting Malaysian silver prices. Price data were collected for one-month, six-month, and twelve-month periods. Prior to model implementation, the data were assessed for normality using QQ-plots and the Jarque-Bera test. Model performance was evaluated using Mean Absolute Percentage Error (MAPE), Symmetric Mean Absolute Percentage Error (sMAPE), and the coefficient of determination (R^2). The results show that both GBM approaches achieved high forecasting accuracy, with MAPE values ranging from 4.9% to 7.5%, sMAPE values ranging from 4.8% to 7.2%, and R^2 values close to 1. However, the log volatility consistently outperformed the simple volatility across all forecasting horizons, producing lower MAPE values and stronger R^2 values. These findings suggest that incorporating log volatility improves the predictive performance of GBM for Malaysian silver prices and highlights its potential as a useful tool for investment decision-making in volatile commodity markets.

Keywords: error metrics, financial forecasting, Monte Carlo simulation, stochastic process, volatility measures.

1. Introduction

The commodities market in Malaysia comprises a variety of real assets, such as agricultural products, industrial metals, and precious metals derivatives. Notably, the commodities market supports the

economic growth of Malaysia and creates jobs; hence, significantly contributing to the nation's gross domestic product. In addition, the diversification of commodities and other real assets can reduce the dependence on a limited number of industries and reduce the impact of economic shocks.

Silver has recently become more affordable, making it more accessible to those with limited funds. Furthermore, silver, known as a precious metal, has high market demand (Gono et al., 2023), particularly due to its significant role in electronic devices. Given that industrial demand forms the foundation for assessing and forecasting fluctuations in silver prices, investors should consider additional factors seriously when making investment decisions. Moreover, silver is often viewed as an alternative to diversification, with its price affected by macroeconomic factors, such as supply and demand, or currency exchange rates.

Additionally, the COVID-19 pandemic raised the demand for silver (Alparslan and Uçar, 2023). Hence, forecasting the value of silver is beneficial to investors. Forecasting is the approach that predicts the direction of commodity prices accurately (Ibrahim & Muda, 2023). These days, many methods have been employed to predict commodity prices, such as Artificial Intelligence (AI) algorithms, contemporary forecasting models, and futures pricing, all of which are highly accurate at predicting commodity prices. Note that commodity price forecasting usually uses futures prices as a reference.

The stages of commodity price forecasting might differ depending on the approach taken. However, several standard steps exist, including calculating return value, estimating parameters, gathering results, calculating stock price projection, and calculating the Mean Absolute Percentage Error (MAPE) value (Ibrahim & Muda, 2023). Thus, this research will provide the basis for a silver price forecast that will include several mathematical expressions frequently used in financial mathematics.

A mathematical model widely used for forecasting is the Geometric Brownian Motion (GBM). Shehzad et al. (2023) overviewed GBM as a popular and straightforward model for simulating stock prices. As one of the most significant and often applied models in financial mathematics, the GBM model can accurately forecast future share prices promptly and is thought to be the fundamental mechanism of a dangerous market (Alhagyan et al., 2017). Additionally, stock markets are commonly assumed to follow Brownian motion, in which asset values fluctuate continuously and often by arbitrary amounts (Mensah et al., 2023; Reddy & Clinton, 2016). Ibrahim et al. (2021) applied GBM and fractional GBM for the simulation of crude palm oil prices. Moreover, Khodayari & Ranjbar (2017) used GBM to solve multidimensional Black-Scholes, and Mehrdoust et al. (2013) used GBM to model multi-asset option price.

The GBM model assumes that the logarithmic returns of the assets follow a normal distribution. Thus, using the quantile-quantile (QQ) plot and the Jarque-Bera (JB) statistical test, this assumption is evaluated to ensure the GBM model is suitable to model prices of silver. The QQ-plot compares the quantiles of the dataset to the quantiles of a theoretical normal distribution (Brys et al., 2004). The data is normally distributed if the data closely follows the diagonal line. The JB test analyzes the skewness and kurtosis of the data to measure how significantly it deviates from a normal distribution (Brys et al., 2004). The data does not significantly deviate from normality if it returns a p-value greater than the 5% significance level.

GBM includes drift and volatility. Volatility is the rate at which the price increases or decreases over a specific period. Typically, volatility signals investors of future fluctuations in its price, and higher volatility indicates higher risk. Volatility is defined as the standard deviation of the random Wiener-driven component (Alhagyan et al., 2017) and price fluctuations are not necessarily linear and proportional (Kayal & Maheswaran, 2021). Hence, this study considered simple volatility and log volatility to obtain the more precise forecast price of silver via GBM.

For the price simulation, we utilized the Monte Carlo simulation which uses randomness to approximate a range of potential outcomes for uncertain events (Nwafor & Oyedele, 2017), a widely used simulation technique widely used in forecasting (Mooney, 1997). Anunak et al. (2023) used the

Monte Carlo simulation to price the options in an extended Heston-CEV hybrid model, while Mehrdoust et al. (2013) employed the Monte Carlo technique with variance reduction to simulate the multi-asset option price.

Various metrics are used to assess the accuracy of future price predictions, such as Mean Absolute Percentage Error (MAPE), symmetric Mean Absolute Percentage Error (sMAPE), and coefficient of determination, R^2 (Chicco et al., 2021). These measures provide valuable insights regarding the effectiveness and the accuracy of the prediction models. While MAPE provides a percentage error that measures accuracy by calculating the average absolute percentage error between actual and forecasted prices (Kim & Kim, 2016), sMAPE adjusts the scale of the data to address the bias for a more balanced and symmetrical measure (Swanson et al., 2011). The coefficient of determination, on the other hand, assesses how closely the predicted prices align with the actual prices. According to Hong and Lee (2005), an R^2 value closer to 1 indicates higher predictive accuracy.

The fluctuations in Malaysia's commodity prices pose challenges for stakeholders, such as traders, investors, and policymakers. Comprehensive studies on the most effective methods for evaluating and predicting these prices variations are essential to understand and manage fluctuations in the silver price. Therefore, this study aims to address this by analyzing two distinct volatility measures in the GBM model using historical data and statistical techniques, enabling stakeholders to make informed decisions.

The main objective of this research is to examine and assess the more accurate volatility measurement for simulating silver prices in Malaysia, using both simple volatility and log volatility within the context of GBM via Monte Carlo simulation. This paper is organized as follows: Section 2 describes the GBM model and its methodology. Section 3 presents the results and discussion, and Section 4 concludes the study with some indication of future work.

2. Materials and Methods

The GBM follows the martingale and Markov characteristics. It states that the predicted value of the random variable S_i, conditional on the whole past event, depends solely on the value of S_{i-1} that came before. The market price S_t, the Wiener process W, the volatility σ, and the drift μ combine to generate the following stochastic model:

$$dS_t = \mu S_t dt + \sigma S_t dW_t. \quad (1)$$

Then, we apply Ito's lemma, which is utilized in one approach for stochastic equations that is an important instrument in the theory of stochastic calculus. According to Ito's Lemma, a function's change over a brief period may be described as the product of two terms: a drift term and a diffusion term. The price of a financial asset might be the function of the problem. While the drift term suggests the function's average rate of change, the diffusion term indicates the change's random component.

Definition (Ito's Lemma) [Øksendal (2003)**].** *In mathematical terms, Ito's Lemma provides that:*

$$d[f(X_t)] = f'(X_t),$$

if X is a stochastic process and $f(X_t)$ is a differentiable function. Then,

$$\frac{1}{2}f''(X_t) + dX_t dt,$$

where $f'(X_t)$ is the first derivative of function f and $f''(X_t)$ is the second derivative, and dX_t is the stochastic component, and dt is the time component.

Let $F_t = \ln(S_t)$ be a function of a log-normal random walk that describes the price process. By applying the Ito's Lemma, and solving the partial derivatives yields:

$$dF_t = \left(\mu - \frac{1}{2}\sigma^2\right)dt + \sigma dW_t.$$

We then integrate Equation (1) to obtain the solution to the stochastic differential equation (SDE) as such:

$$S_t = S_0 e^{\left(\mu - \frac{1}{2}\sigma^2\right)t + \sigma W_t}, \quad (2)$$

which is fundamental in the simulation of the asset prices, where the values of μ and σ are calibrated using historical logarithmic returns of the commodity. Taking logarithms from Equation (2) shows that $\ln S_t$ is normal with mean $\mu t + \ln S_0$, and variance $\sigma^2 t$; hence, for each t, S_t is the silver price at time t that has a log-normal distribution. The simulation of the prices uses Equation (2) where we first estimate the mean, μ, and volatility, σ.

2.1 Estimation of the Drift and Standard Deviation

Prior to trading resuming the next trading day, the closing price reflects the most recent and closed valuation of stocks or indexes. Investing money in stocks over time is uncertain due to the closing price volatility. The values of the daily closing prices may be translated into a daily return using the formula below:

$$R_i = \ln\left(\frac{S_i}{S_{i-1}}\right),$$

for $i \geq 1$, where R_i is the logarithmic return of the commodity prices, S_i is the commodity price on day i (that day), and S_{i-1} is the commodity price one day before day i (the previous day). For N number of time periods in each sample, the mean of the logarithmic returns, μ is computed as follows:

$$\mu = \frac{1}{N}\sum_{i=1}^{N} R_i, \quad (3)$$

while the standard deviation, or the volatility, is computed using two different volatility measurements, which are simple volatility, σ_{sim}, and log volatility, σ_{log}, respectively as follows:

$$\sigma_{sim} = \sqrt{\frac{1}{N-1}\sum_{i=1}^{N}(R_i - \mu)^2}, \quad (4)$$

$$\sigma_{log} = \sqrt{\frac{1}{N-1}\sum_{i=1}^{N} R_i^2}. \quad (5)$$

Using a sample of silver prices, we demonstrate the calculation of the logarithmic returns, drift, and standard deviation, as documented in Table 1.

Table 1. Estimation of the drift and volatility for a sample of silver prices of 5 days.

$\boldsymbol{\mu}$	$\boldsymbol{\frac{1}{5}\sum_{i=1}^{5} R_i = 0.00267}$
$\boldsymbol{\sigma_{sim}}$	0.003195
$\boldsymbol{\sigma_{log}}$	0.005457

2.2 Accuracy Measurement

To measure the accuracy of the forecasts produced by each volatility measurement, we computed the MAPE, sMAPE, and R^2 (Chicco et al., 2021). The accuracy measurements, ϵ, and R^2, are defined as such:

$$\epsilon_{MAPE} = \frac{1}{N}\sum_{i=1}^{N}\left|\frac{S_i - P_i}{S_i}\right| \times 100\%,$$

$$\epsilon_{sMAPE} = \sum_{i=1}^{N} \frac{|P_i - S_i|}{\left[\frac{|P_i|+|S_i|}{2}\right]} \times 100\%,$$

$$R^2 = 1 - \frac{\sum_{i=1}^{N}(P_i - S_i)^2}{\sum_{i=1}^{N}(\mu - S_i)^2},$$

where N is the number of time periods, S_i is the actual (historical) silver price at day i, P_i is the forecast silver price, and μ is the mean of the returns.

3. Results and Discussion

This section presents a comprehensive analysis of the data considered in the study[1]. The data is the closing prices for Malaysian silver that was collected from Exchange Rates (2024) for three periods: one-month, six-month, and twelve-month, excluding non-trading days, weekends, and public holidays. The one-month period had 23 observations from May 1, 2023, until May 31, 2023; the six-month period had 132 observations from May 1, 2023, until October 31, 2023; and the twelve-month period had 262 observations from May 1, 2023, until April 30, 2024.

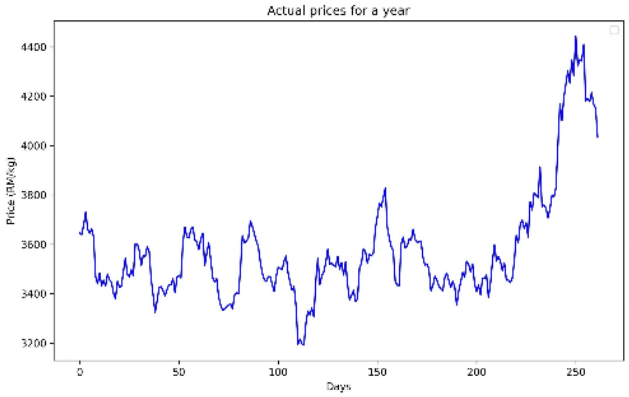


(a) One-month



(b) Six-month

[1] The analysis was done using Microsoft Excel, a cost-effective and widely available tool. No specialized hardware was needed, making the method practical for general users and analysts without advanced computing resources.



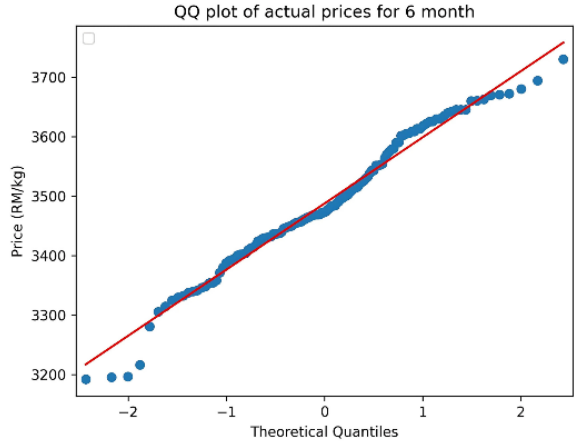
(c) Twelve-month

Figure 1. The actual price for three simulation periods: one-month, six-month, and twelve-month.

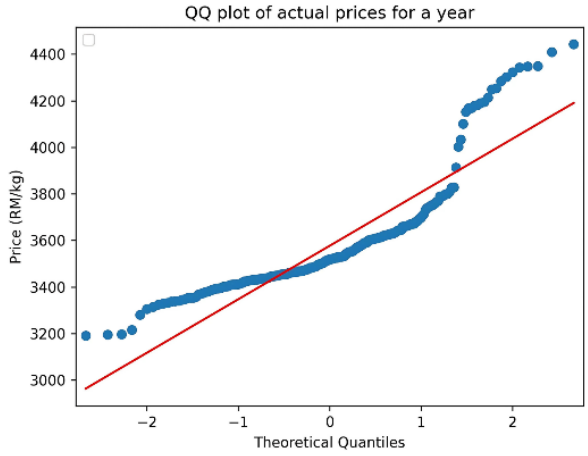
The simulation periods are the forecasting horizon for the one-month, six-month and twelve-month datasets. The actual three simulation periods are plotted in Figure 1. Based on Figure 1(a), the data show that the early month had a higher silver price but started to decrease, and the fluctuation remained from day 9 to the end of the month. Meanwhile, the six-month series in Figure 1(b) shows some seasonality, indicating more volatility within this period. The series is now obviously volatile in Figure 1(c). The data fluctuate around RM3200/kg to RM3700/kg until day 210 but increases to RM4400/kg. This suggests that silver prices are more stable in the short term and become more unpredictable and volatile over longer periods.



(a) One-month



(b) Six-month



(c) Twelve-month

Figure 2. QQ-plot for the actual price for three simulation periods: one-month, six-month,and twelve-month.

Moreover, we plot the QQ-plot and perform the Jarque-Bera test to evaluate whether the data follows a normal distribution, a key assumption when applying the GBM model to simulate and forecast silver prices. The QQ-plots are displayed in Figure 2 for one-month, six-month, and twelve-month periods and show that most data points align closely with the diagonal line, indicating a good fit to a normal distribution, especially in the middle range of prices. However, consistent deviations at the tails across all periods suggest that extreme values may not follow a normal distribution. This pattern could indicate the presence of outliers or a distribution with heavier tails than the normal distribution.

In addition to the QQ-plots, the JB hypothesis testing is conducted for all three data sets. The JB test was used to determine whether the actual silver prices are normally distributed, with the following hypothesis:

H_0: Actual silver prices are normally distributed.

H_1: Actual silver prices are not normally distributed.

From Table 2, the p-values for one-month and six-month are greater than the significance level of 5% (0.05). Hence, the null hypothesis cannot be rejected, suggesting that the actual silver prices are normally distributed for the one-month and six-month periods. For the twelve-month period, the null hypothesis is rejected, indicating deviation from normality.

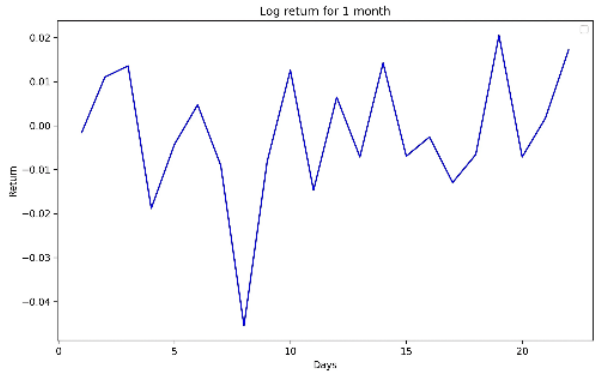
Table 2. Jarque-Bera (JB) test of actual silver prices: one-month, six-month and twelve-month.

Description	One-month (1 May–31 May 2023)	Six-month (1 May–31 Oct 2023)	Twelve-month (1 May 2023–30 Apr 2024)
Number of observations	23	132	262
Sample skew	0.62137	-0.21699	1.86849
Sample kurtosis	-1.30723	-0.08915	3.64119
JB test statistics	3.11770	1.07957	297.18770
p-value	0.21038	0.58287	2.9276×10^{-65}

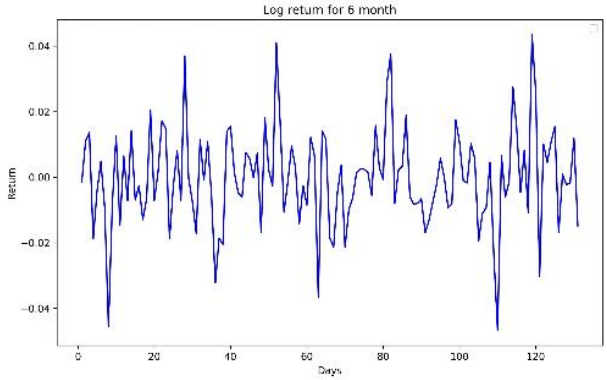
Table 3. Jarque-Bera (JB) test of logarithmic returns of silver prices: one-month, six-month and twelve-month.

Description	One-month (1 May–31 May 2023)	Six-month (1 May–31 Oct 2023)	Twelve-month (1 May 2023–30 Apr 2024)
Number of observations	22	131	261
Sample skew	-0.97334	-0.00028	-0.00342
Sample kurtosis	2.37167	1.25495	1.05156
JB test statistics	8.62988	8.59639	12.02591
p-value	0.01337	0.01359	0.00245

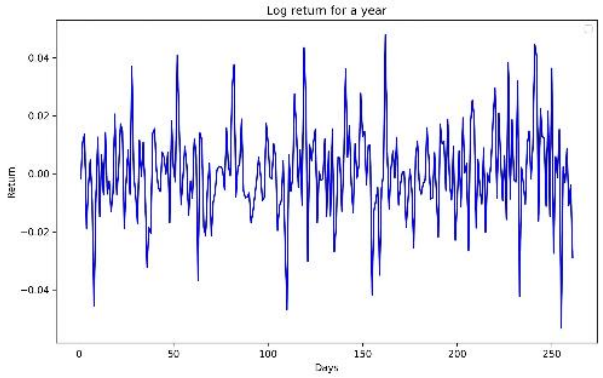
The silver price, hence, transforms and uses logarithmic returns to normalize the datasets. Since deviations are observed in Figure 2, the transformation includes the twelve-month of actual silver price data.



(a) One-month



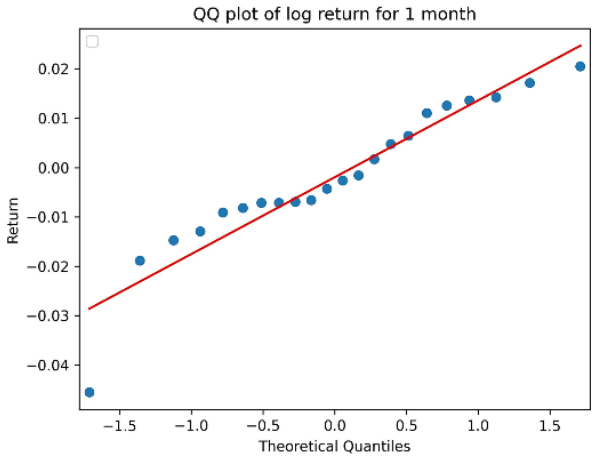
(b) Six-month



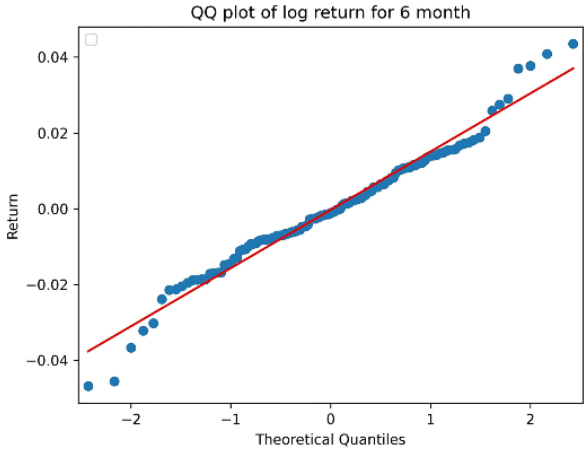
(c) Twelve-month

Figure 3. The logarithmic returns of the actual price for three simulation periods: one-month, six-month, and twelve-month.

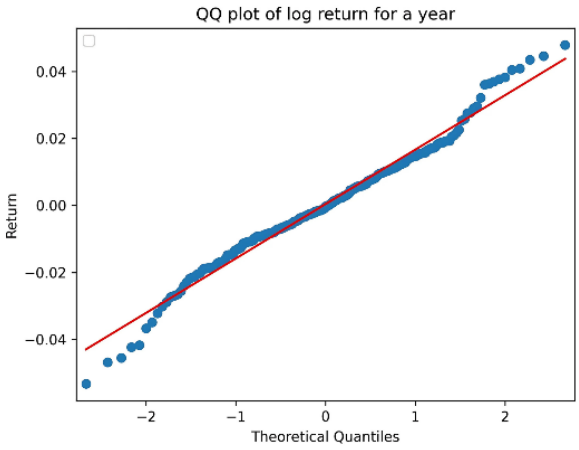
GBM assumes normally distributed logarithmic returns, which implies lognormally distributed prices (Hull, 2014). The logarithmic returns are shown in Figure 3, and the normality is further validated using QQ-plots, as shown in Figure 4. The logarithmic return plots in Figure 3 support a pattern of increased return volatility over an extended period. As time grows, silver returns become less regularly distributed and more volatile, as shown by the more noticeable spikes in the return graphs for the six-month and twelve-month periods.



(a) One-month



(b) Six-month



(c) Twelve-month

Figure 4. QQ-plot for the logarithmic returns of the actual price for three simulations periods: one-month, six-month, and twelve-month.

Based on QQ-plots in Figure 4, the data is approximately normally distributed across all three periods, supporting the forecast ability and the use of the GBM model in this study. Logarithmic returns, especially for the one-month and six-month periods, exhibit a strong fit to the normal distribution, with most points closely following the diagonal line and only minor deviations. This trend continues with the twelve-month of logarithmic returns, confirming that logarithmic returns across all periods align well with the normality assumption required for the GBM model. Moreover, we perform a JB test on the logarithmic returns for the hypotheses below, and the results are documented in Table 3:

H_0: Logarithmic returns of silver prices are normally distributed.

H_1: Logarithmic returns of silver prices are not normally distributed.

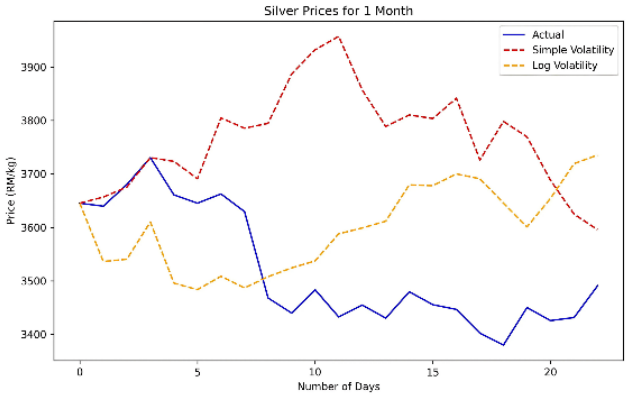
Since the p-values are less than the significance level of 0.05 for all periods, the null hypothesis is rejected (see Table 3). Nevertheless, the corresponding QQ-plots indicate that the logarithmic returns approximately follow a normal distribution, particularly in the central region. Since GBM is widely applied in financial modelling under approximate normality assumptions, the model was retained for forecasting, where the parameters μ, σ_{sim}, and σ_{log} are estimated to simulate the projected silver prices using Equations (3), (4) and (5), respectively. The estimated values are tabulated in Table 4. The Monte Carlo simulation technique is used as the simulation method to simulate the silver prices following

standard practice in financial simulation (Glasserman, 2004). The presence of a random value parameter requires the application of Monte Carlo simulation (Xiang et al., 2021). The simulation is applied to Equation (2) to forecast the silver prices. This study used Microsoft Excel to implement the Monte Carlo simulation.

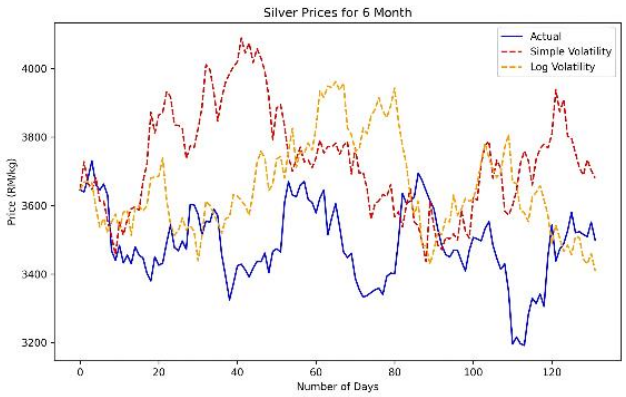
Table 4. Estimation of mean and volatility: one-month, six-month and twelve-month.

Period	One-month	Six-month	Twelve-month
$\boldsymbol{\mu}$	-0.00197	-0.00031	0.00039
$\boldsymbol{\sigma_{sim}}$	0.01411	0.01515	0.01749
$\boldsymbol{\sigma_{log}}$	0.01485	0.01516	0.01615

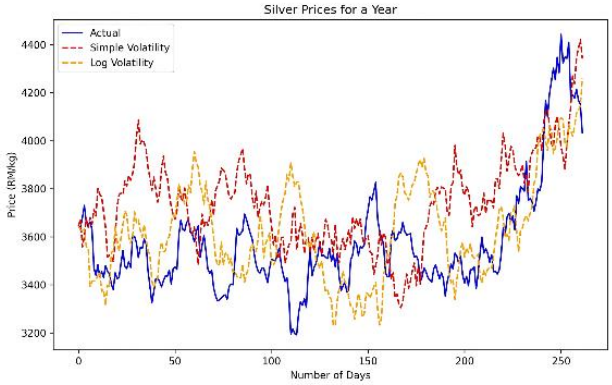
Figures 5(a), 5(b), and 5(c) illustrate that the forecast line based on log volatility is closer to the actual price line compared to the forecast line based on simple volatility. This suggests that the log volatility model is more accurate in predicting silver prices. Across all periods, the log volatility prediction consistently aligns with the actual prices more accurately than the simple volatility. This suggests that the log volatility model is better than the simple volatility model in reflecting the price changes of silver. The log transformation helps to stabilize variance and manages the intrinsic characteristics (skewness or heavy tails) of silver prices more effectively. Table 5 documents the accuracy assessment of this model across three different simulation periods (one month, six months, and twelve months) employing simple volatility and log volatility, while measuring MAPE, sMAPE, and R^2.



(a) One-month



(b) Six-month



(c) Twelve-month

Figure 5. Simulation of silver prices for three simulation periods: one-month, six-month, and twelve-month.

Table 5. Accuracy measurement of the forecasting model.

	Period	One-month	Six-month	Twelve-month
MAPE	σ_{sim}	7.40478	7.56009	6.45201
	σ_{log}	4.92717	5.78950	4.93785
sMAPE	σ_{sim}	7.04075	7.17215	6.23293
	σ_{log}	4.83730	5.55504	4.86129
$\boldsymbol{R^2}$	σ_{sim}	0.99329	0.93955	0.99487
	σ_{log}	0.99747	0.96269	0.99654
Average Time		*8 sec*	*22 sec*	*39 sec*

According to Chicco et al. (2021), the range of sMAPE is from 0% to 200%, and lower sMAPE values indicate more accurate forecasts. For R^2, the range starts with 0% to 1%, where higher R^2 values indicate better fit and more accurate predictions. Table 5 shows that the log volatility model outperforms the simple volatility model in predicting silver prices across all periods. The log model's lower MAPE and sMAPE values signify higher accuracy and reliability. Conversely, the higher values of R^2 indicate a more accurate alignment of the predicted prices to the actual prices. This consistent performance indicates that the log volatility model is better at capturing the complexity of the silver prices.

4. Conclusion

This study evaluates two types of volatility, simple and log volatility, calculated from historical silver price data. The Monte Carlo simulation was employed to predict Malaysia's silver prices across three periods: one month, six months, and twelve months. The accuracy of the model was evaluated using MAPE, sMAPE, and R^2. Every simulation generated MAPE and sMAPE results lower than 10%, with R^2 consistently close to 1, demonstrating highly accurate strong prediction. Log volatility outperformed simple volatility, attaining MAPE values ranging from 4.9% to 5.8%, with a 2.6% improvement. While remaining simple and accessible, our findings show higher accuracy compared to Gono et al. (2023), who reported a MAPE of 6.06% using XGBoost.

These findings emphasize the GBM model with log volatility as a reliable and economical prediction tool for investors and analysts. This study emphasizes in-sample performance and the model's ability to fit historical data. The results provide evidence that GBM with log volatility is an

effective tool for predicting silver prices, and it offers valuable insights into research and application in commodity markets.

Future studies might investigate longer-term forecasts and utilize the model for different commodities such as palm oil and copper. Additionally, the modelling of commodity prices can be evaluated using more advanced models like the Heston model to more accurately reflect stochastic volatility.

5. Acknowledgements

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