

## LEARNING NURBS THROUGH S-CAD

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### ABSTRACT

*S-CAD or Simple Computer Aided Design is a CAD software specially design and develop for Senior High School student introduced by the CAD Research Group, Universiti Kebangsaan Malaysia. The software provides most of the functions normally available in CAD systems. This paper introduces the NURBS entity, e.g., the degree value, the knot vector, the control points and weight of control point. Consequently the paper discusses the learning and understanding of this entity using the S-CAD.*

**Keywords:** *NURBS, knot vector, control point, weight of control point*

### 1.0 INTRODUCTION

A CAD software is a software normally used for design activity. Most of the software available in the market are for professional and the price is very prohibitively expensive and require high-end hardware.

For that reason, a group of researchers from UKM has developed software called S-CAD which run on DOS. The software provides most of the CAD functions such as line, curve and spline. The polygon and the object (2D and 3D) can be manipulated using functions such as mirror transformation, trimming, scaling, translating, flipping, copying and etc. S-CAD also has the User Coordinate System (UCS).

Most inputs in the system uses mouse. Thus designing activity is using mouse and sometimes keyboard (for saving, etc.). The software uses a pull-down menu and icons. The minimum hardware requirements are a 386 based PC, a mouse, a keyboard and one MB RAM. The first version of S-CAD is already installed at 114 secondary schools and 31 teacher training colleges throughout Malaysia in January 1997.

This paper describes how Non Uniform Rational B-Spline (NURBS) entities [1]; the degree value, the knot vector, the control points and the weight of control point can be learned using S-CAD.

### 2.0 NURBS CURVE

The theoretical background of NURBS used in S-CAD was discussed in many papers [2,3,4,5,6]. NURBS curve is a piecewise rational polynomial function of the form:

$$C(u) = \frac{\sum_{i=0}^n w_i P_i N_{i,p}(u)}{\sum_{i=0}^n w_i N_{i,p}(u)} \dots\dots\dots (2.1)$$

where n is the number of control points,  $w_i$  is the weight of control points,  $P_i$  is the control points, and  $N_{i,p}(u)$  are the p th-degree B-spline basis functions on U direction defined recursively as [7]:

$$N_{i,0}(u) = \begin{cases} 1 & u_i < u \leq u_{i+1} \\ 0 & \text{otherwise} \end{cases} \dots\dots\dots (2.2a)$$

$$N_{i,p}(u) = \frac{u - u_i}{u_{i+p} - u_i} N_{i,p-1}(u) + \frac{u_{i+p+1} - u}{u_{i+p+1} - u_{i+1}} N_{i+1,p-1}(u) \dots\dots\dots (2.2b)$$

where  $u_i$  are knot vectors:  $U = \{ u_0, u_1, \dots, u_m \} \dots\dots (2.3)$

The number of knot vector is dependent on the degree of polynomial (p) and the number of control points (n) with  $m = n + p + 1$ . The different categories of B-Spline, that are related to knot vector [1], are the Uniform B-Spline, the Open Uniform B-Spline (with standard knot vector) and the Non Uniform B-Spline.

The Uniform B-Spline is the B-Spline function with constant knot spacing. For example:  $n=4, p=3$  (cubic) and

the knot spacing =1. Thus the knot vector **U** is {1, 2, 3, 4, 5, 6, 7, 8}.

The Open Uniform B-Spline is the B-Spline function with the knot spacing as a combination of Uniform B-Spline and Non Uniform B-Spline. This knot vector will become standard knot vector if all the knot spacing is uniform except knot spacing at both end which are repeated (p+1) times.

Example:

$$U = \{0, 0, 1, 2, 3, 3\}, \quad p = 1 \text{ and } n = 4$$

$$U = \{0, 0, 0, 0, 1, 2, 2, 2, 2\}, \quad p = 3 \text{ and } n = 5$$

The Non Uniform B-Spline is the B-Spline function with non uniform knot spacing except at both ends which are repeated (p+1) times. For example: p=2 and n=7, the knot vector is **U** = {0, 0, 0, 1, 2, 2, 3, 4, 4, 4}. Therefore NURBS is a piecewise rational polynomial B-Spline function that it's knot vector is non uniform.

### 3.0 NURBS SURFACE

The NURBS surface is given by the equation [6]:

$$S(u, v) = \frac{\sum_{i=0}^n \sum_{j=0}^m \omega_{i,j} P_{i,j} N_{i,p}(u) \cdot N_{j,q}(v)}{\sum_{i=0}^n \sum_{j=0}^m \omega_{i,j} N_{i,p}(u) \cdot N_{j,q}(v)} \dots\dots\dots (3.0)$$

where

- u, v** the surface's parameter
- N<sub>i,p</sub>(u)** and **N<sub>i,q</sub>(v)** the normalized B-spline basis functions with degree p and q
- p, q** degrees in u and v direction
- P<sub>i,j</sub>** the mesh of control points with i=0...n and j=0...m.
- ω<sub>i,j</sub>** the weight of control points

### 4.0 LEARNING NURBS'S PROPERTIES THROUGH S-CAD

NURBS's properties can be learned very easily through S-CAD. Users only use mouse to enter any number, for example, to change the value of weight of control point, degree of curve (or surface), and knot vector. In addition, user can make any modifications to the curve's (or surface's) shape freely and interactively.

The S-CAD systems will enable users to see results immediately after making changes and modifications. Thus will help users to learn NURBS's properties through observation.

Before using the S-CAD, user has to know its default value. The default value of NURBS is Open Uniform

quadratic B-Spline. It's mean that the weight of control point is one, the degree of polynomial is two and the knot vector is standard knot vector.

- **The control point.**

Control points for curve (surface) are the set of points representing B-Spline. The control points are not the points on the curve (surface) but their positions will be able to determine the shape of curve (surface). Fig. 1.b. shows the resulting curve if point **P<sub>1</sub>** (Fig. 1.a.) is changed to **P<sub>1'</sub>**. Furthermore, Fig. 1.d. shows the resulting surface if the points **P<sub>0,1</sub>**, **P<sub>1,1</sub>**, **P<sub>2,1</sub>**, **P<sub>3,1</sub>**, **P<sub>4,1</sub>**, **P<sub>5,1</sub>**, and **P<sub>6,1</sub>** (Fig. 1.c) are changed to the new location.

- **The degree value:**

The degree value has impact on the shape of curve (and surface). For example, conic (ellipse, parabolic, hyperbolic) functions are quadratic functions or the degree value is two. Fig. 2 shows the effect of changing from Open Uniform **quadratic** B-Spline (Fig. 1) to Open Uniform **cubics** B-Spline.

- **The knot vector.**

From the knot vector, user will be able to learn the categories of B-Spline and its impact to the curve's (surface's) shape. In general, different category of B-Spline will result in different curve's (surface's) shape. For example, using similar control points, Uniform B-Spline curve with knot vector {0,1,2,3,4,5,6,7}, and Non Uniform B-Spline curve with knot vector {0,0,0,2,5,7,7,7} are obtained (Fig. 3.a and Fig. 3.b).

- **The weight of control point.**

The geometric meaning of the weight of control point and its analysis has been discussed extensively by L. Piegel [4,5,6]. This paper will only show how users can practically learn the weight of control point using the S-CAD. For example, Fig. 4.a. shows that ω<sub>1</sub> = 1 (see Fig. 1.a) is changed to ω<sub>1</sub> = 5 and the curve is pushed toward point **P<sub>1</sub>**. Furthermore, Fig. 4.c. shows that ω<sub>4,1</sub> = 1 (from Fig. 4.b) is changed to ω<sub>4,1</sub> = 4 and the resulting of surface is pushed toward point **P<sub>4,1</sub>**.

### 5.0 CONCLUSION

By using S-CAD, users can learn the NURBS's properties more easily through observation. They can make modifications of the shape of curve (surface) freely and interactively by changing the control points, the degree of the polynomials, the knot vector and the weight of control point.

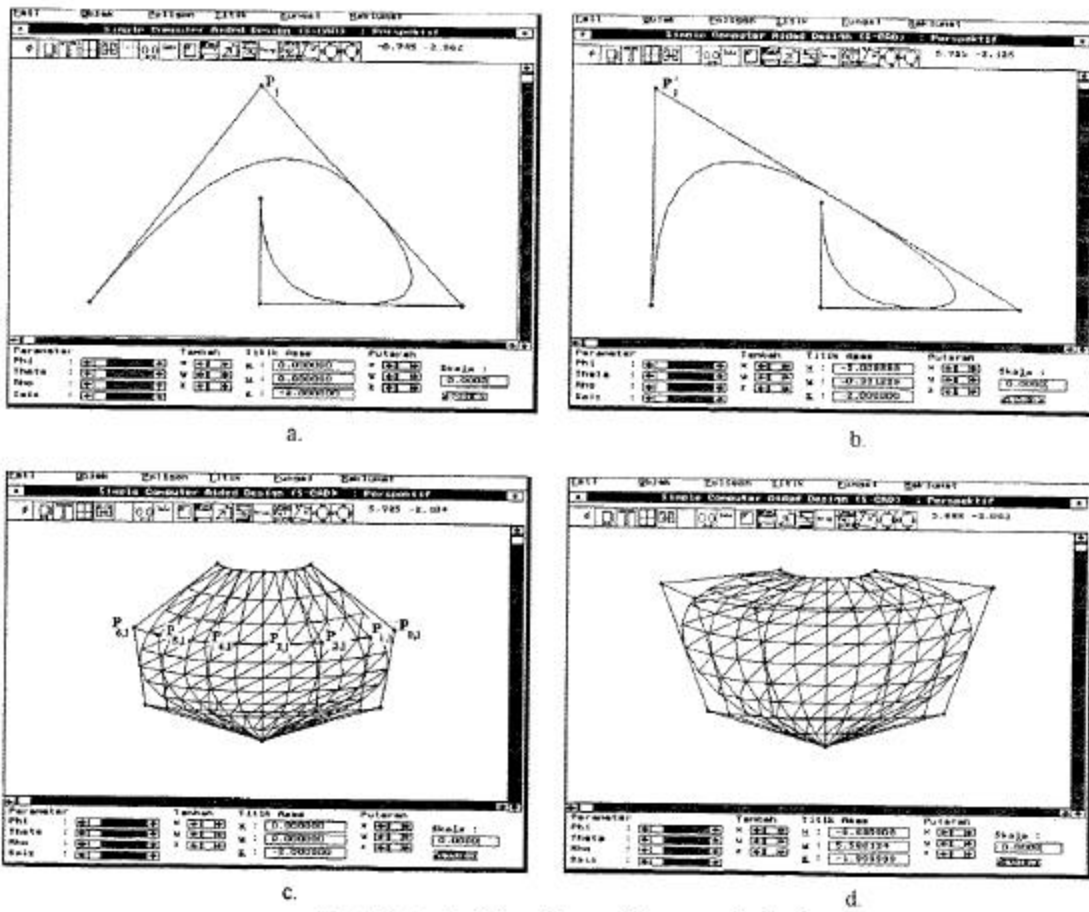


Fig. 1: Control point and the resulting curve (surface).

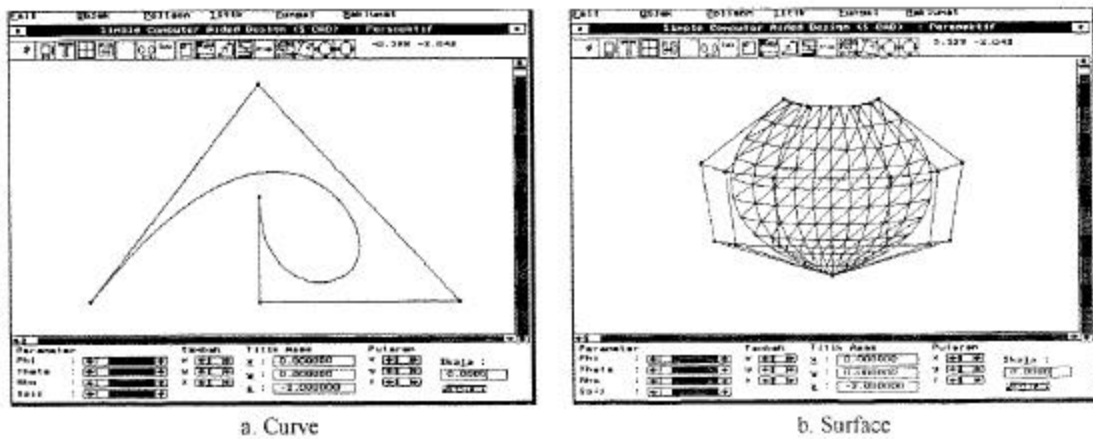


Fig. 2: The Open Uniform Cubic B-Spline

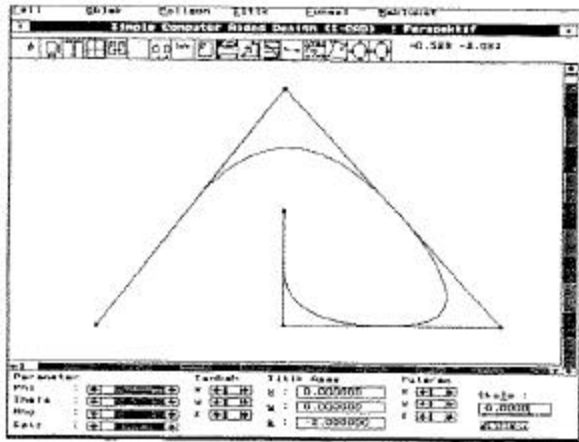


Fig.3.a: Uniform B-Spline

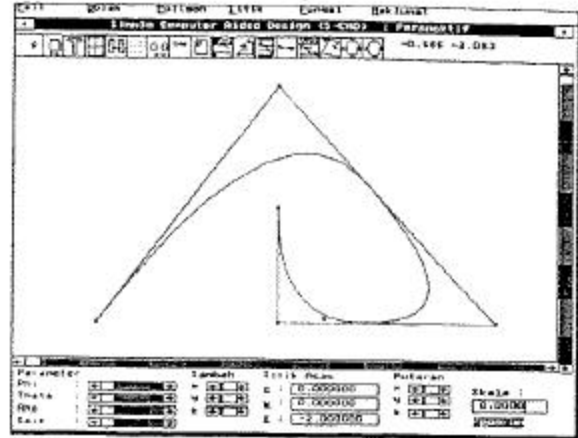
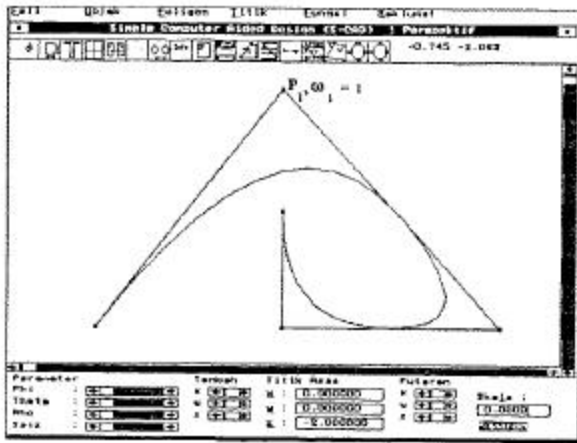
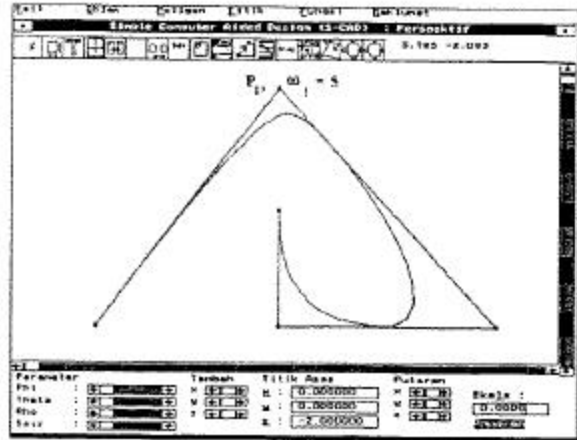


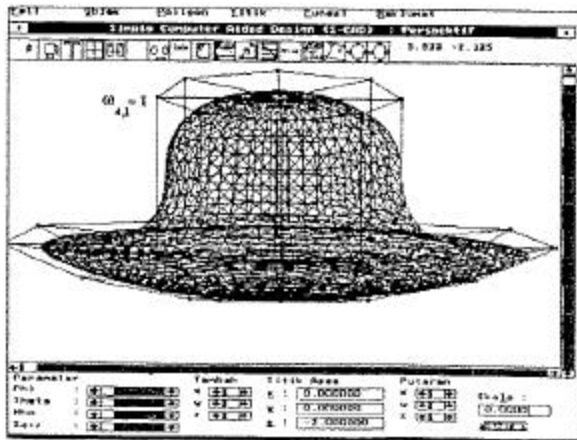
Fig.3.b: Non Uniform B-Spline.



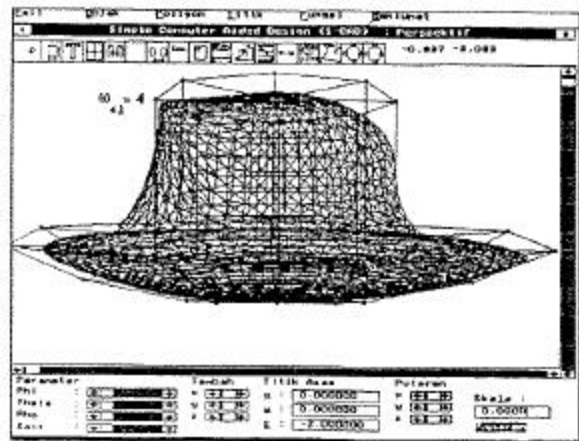
a.  $P_1$  with  $\omega_1 = 1$ .



b.  $P_1$  with  $\omega_1 = 5$ .



c.  $P_{4,1}$  with  $\omega_{4,1} = 1$ .



d.  $P_{4,1}$  with  $\omega_{4,1} =$

Fig. 4: The effect of changing the weight ( $\omega$ ) of control point

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## BIOGRAPHY

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