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USING THE TWO-DIMENSIONAL TRANSMISSION LINE MODELLING TECHNIQUE TO DESIGN LADDER WAVE DIGITAL FILTERS

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ABSTRACT

Extends the time domain design approach first introduced for cascaded unit element wave digital filters (WDFs) to ladder WDFs. It shows the correspondence between ladder WDFs and the two-dimensional transmission line networks. The transmission line modelling technique facilitates the design and optimization of ladder WDFs by affording a convenient method of calculating the sensitivities with respect to the design parameters.

Keywords: ***Optimization, Transmission line modelling, Wave digital filter***

1.0 INTRODUCTION

Wave digital filters [1] (WDFs) emulate classical analogue filters, having structures that are similar to their analogue counterparts. Furthermore, WDFs carry over through to the digital domain the sensitivity properties of their reference analogue filters. Analogue filters with resistive terminations are known for their high tolerance to element value variation [2]. WDFs exhibit this tolerance by being highly insensitive to the coefficient value variation. This results in filters that can be realized with shorter coefficient wordlengths compared to other types of digital filters.

The short wordlength feature has resulted in a lot of interest in designing WDFs [1]. One approach is the time domain design based on the transmission line modelling (TLM) technique. The TLM technique [3, 4] is a numerical analysis technique capable of solving partial differential equations. Physical phenomena that can be described with partial differential equations have been successfully solved by the TLM technique. The TLM technique is based on discretizing partial differential equations in both time and space. These equations are discretized in space by modelling them with electrical circuits. Discretization in time is afforded by modelling the electrical circuits with transmission lines to produce the equivalent transmission line models. Electrical circuits which are describable with differential equations have also been successfully modelled and designed using TLM [5, 6].

The TLM technique has been shown to be suitable for the time domain design of cascaded unit element WDFs [7]. The correspondence between a cascaded unit element WDF and the one-dimensional TLM algorithm has been shown. This paper extends the time domain design approach to ladder WDFs. Cascaded unit element WDFs are modelled after distributed unit element filters while ladder WDFs are modelled after ladder analogue filters [8]. Structures of the two types of WDFs differ reflecting the differing configurations of their reference filters. Unlike cascaded unit element WDFs, two-dimensional transmission line networks are required to model the ladder WDFs.

2.0 LADDER WDFS

There are many types of WDFs reflecting the many varieties of analogue filters from which they are derived. Ladder WDFs are derived from LC ladder analogue filters (Fig. 1).

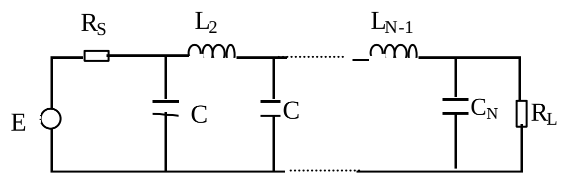

Fig. 1: Analogue Ladder Filter

Ladder WDFs mimic the structure of ladder analogue filters. These WDFs consist of three-port adaptors that house the digital adders and multipliers. The adaptors simulate the series and parallel connections of the inductors and capacitors of the analogue filters.

In theory, it is possible to realize ladder WDFs by directly interconnecting the three-port adaptors [8]. However, in practice, a more practical approach to implementation is to place delay elements in between the adaptors [9]. This requires some modifications on the reference ladder filters, where unit elements (UEs) are inserted and shifted to the appropriate places using Keroda's transformation [10]. Fig. 2a shows the modified analogue ladder filter while Fig. 2b shows the corresponding ladder WDF, which are realized with three-port parallel adaptors.

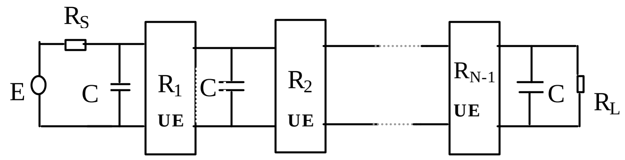

Fig. 2a: Modified Analogue Ladder Filter

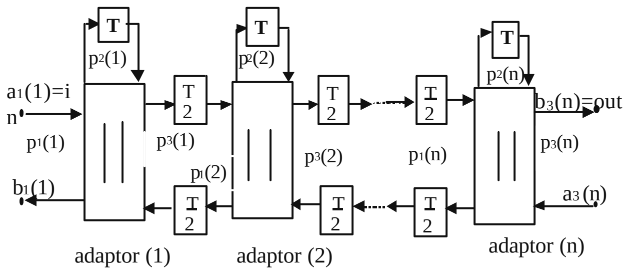

Fig. 2b: Ladder WDF with Parallel Adaptors

Ladder WDFs can be realized with three-port series adaptors by suitably modifying the reference analogue filters. Fig. 3a shows the modifications required while Fig. 3b shows the corresponding ladder WDF.

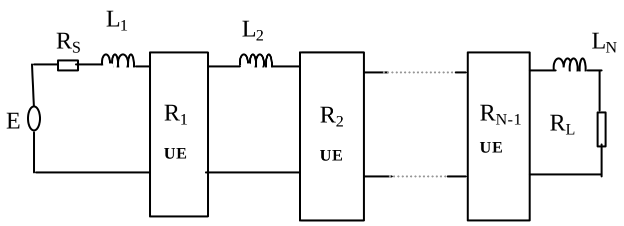

Fig. 3a: Modified Analogue Ladder Filter

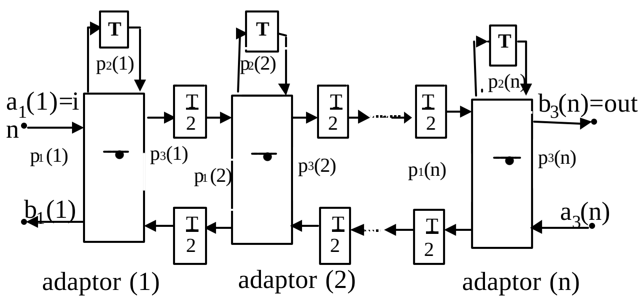

Fig. 3b: Ladder WDF with Series Adaptors

At a sampling frequency F_s, the sampling period T is defined as

$$T = \frac{1}{F_s} \tag{1}$$

The equations that describe the operations of the adaptors relate the output to the input signals. Utilizing the principle of port dependence[1], the equations for the parallel three-port adaptor with the third port as the dependent port are

$$\begin{aligned} b_3 &= a_3 - \sum_{p=1}^{2} g_p (a_3 - a_p) \\ b_p &= b_3 + (a_3 - a_p), \quad p = 1,2 \end{aligned} \tag{2}$$

where a_p represents the input signal, b_p the output and g_p the multiplying coefficient associated with port p.

In a similar fashion, with the third port selected as the dependent port, the equations that describe the series three-port adaptors are

$$\begin{aligned} b_3 &= -(b_1 + b_2 + \sum_{p=1}^{3} a_p) \\ b_p &= a_p - g_p \sum_{p=1}^{3} a_p, \quad p = 1,2 \end{aligned} \tag{3}$$

According to equations (2) and (3), two multiplying coefficients are required for each adaptor.

3.0 TLM EQUIVALENCE

The TLM technique is based on modelling using transmission line networks. The transmission line models are uni- or multidimensional depending on the complexity of the entities that they model. The uni-dimensional model also known as the link model has been shown to be equivalent in structure to the cascaded unit element WDF [7]. A ladder WDF has a structure that is similar to the two-dimensional mixed transmission line model. The model consists of interconnecting equal length transmission lines and stubs. The adaptors of the WDFs are represented by the junctions of the transmission lines. Fig. 4(a) shows a mixed transmission line model equivalent to a ladder WDF with parallel adaptors, while Fig. 4(b) corresponds to that with series adaptors.

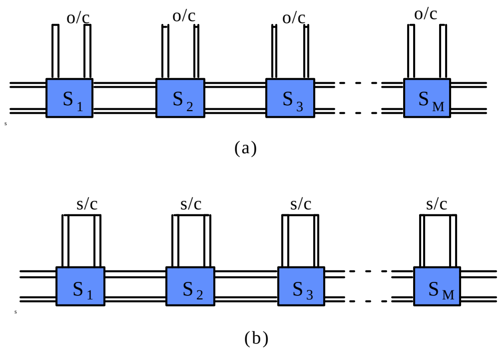

Fig. 4: TLM Mixed Models. (a) Structure is equivalent to a ladder WDF with parallel adaptors. (b) Structure is equivalent to a ladder WDF with series adaptors.

Adopting the transmission line algorithm makes it possible to design ladder WDFs in the time domain. A pulse that travels down the length of a transmission line will experience the scattering phenomenon when it reaches a junction. Part of the pulse is transmitted to the adjacent transmission line while the rest is reflected back to the original line. If T_d denotes the time that it takes for a pulse to travel down one length of a transmission line, it is related to the sampling frequency, F_s, of the ladder WDF according to

$$T_d = \frac{1}{2F_s} \tag{4}$$

At the *m*th junction the three incident, *a*, and reflected pulses, *b*, can be represented by the matrices

$$\mathbf{a}_m = \begin{bmatrix} a_{m1} \\ a_{m2} \\ a_{m3} \end{bmatrix}, \quad \mathbf{b}_m = \begin{bmatrix} b_{m1} \\ b_{m2} \\ b_{m3} \end{bmatrix} \tag{5}$$

The scattering equation for the *m*th junction is

$$\mathbf{b}_{m,k} = \mathbf{S}_m \mathbf{a}_{m,k} \tag{6}$$

where k is the kth time step and **S** the scattering matrix of the junction.

For the TLM network, the **S** matrix is dependent on the characteristic impedance of the transmission lines. In modelling a ladder WDF, the equivalent **S** matrix can be derived from the equations (2) and (3) describing the adaptors. The coefficients associated with the *m*th junction can be written as the vector

$$\mathbf{g}_m = [g_1, g_2] \tag{7}$$

For a three-port parallel adaptor, the **S** matrix at the *m*th junction becomes

$$\mathbf{S}_m = \begin{bmatrix} g_1 - 1 & g_2 & 2 - g_1 - g_2 \\ g_1 & g_2 - 1 & 2 - g_1 - g_2 \\ g_1 & g_2 & 1 - g_1 - g_2 \end{bmatrix} \tag{8}$$

while for the three-port series adaptor, this matrix can be shown to be equal to

$$\mathbf{S}_m = \begin{bmatrix} 1 - g_1 & -g_1 & -g_1 \\ -g_2 & 1 - g_2 & -g_2 \\ g_1 + g_2 - 2 & g_1 + g_2 - 2 & g_1 + g_2 - 1 \end{bmatrix} \tag{9}$$

The entire transmission line network is made up of M junctions. The coefficient vector for the entire network can be written as

$$\mathbf{g} = [\mathbf{g}_1, \mathbf{g}_2, \ldots\ldots, \mathbf{g}_M] \tag{10}$$

The scattering equation for the entire network is

$$\mathbf{b}_k = \mathbf{S}\mathbf{a}_k \tag{11}$$

where **S** is now a block diagonal partitioned matrix with $\mathbf{S}_1$, $\mathbf{S}_2$,.....,$\mathbf{S}_m$ on the diagonal, while **a** and **b** are the collection of all incident and reflected pulses assembled into the partitioned vectors

$$\mathbf{a} = \begin{bmatrix} \mathbf{a}_1 \\ \mathbf{a}_2 \\ \vdots \\ \mathbf{a}_M \end{bmatrix}, \quad \mathbf{b} = \begin{bmatrix} \mathbf{b}_1 \\ \mathbf{b}_2 \\ \vdots \\ \mathbf{b}_M \end{bmatrix} \tag{12}$$

The propagation of the reflected and incident signals during delay T_d can be described by the connection matrix **C**. However, since it takes $2T_d$ for a pulse to travel down the length of a transmission line stub and back again, a connection matrix **C'** is required to indicate this property. For the same filter order, the **C** matrix is equivalent for both the parallel and series adaptor configurations. However, the non-zero entries for the matrix **C'** are all 1 for the parallel and -1 for the series adaptors. The connection equation for the transmission line network can be written as

$$\mathbf{a}_{k+1} = \mathbf{C}\mathbf{b}_k + \mathbf{C}'\mathbf{b}_{k-1} \tag{13}$$

A time domain design using the gradient based optimization algorithm requires the calculations of the first and second order sensitivities with respect to the design variable. The equations of the first order sensitivities with respect to the *j*th design parameter for ladder WDFs are found by differentiating equations (11) and (13) with respect to g_j. The resulting equations are

$$\frac{\partial \mathbf{b}_k}{\partial g_j} = \frac{\partial \mathbf{S}}{\partial g_j}\mathbf{a}_k + \mathbf{S}\frac{\partial \mathbf{a}_k}{\partial g_j} \quad (14)$$

and

$$\frac{\partial \mathbf{a}_{k+1}}{\partial g_j} = \mathbf{C}\frac{\partial \mathbf{b}_k}{\partial g_j} + \mathbf{C'}\frac{\partial \mathbf{b}_{k-1}}{\partial g_j} \quad (15)$$

Consequently, the second order sensitivities with respect to the lth and jth design parameters are

$$\frac{\partial^2 \mathbf{b}_k}{\partial g_l \partial g_j} = \frac{\partial^2 \mathbf{S}}{\partial g_l \partial g_j}\mathbf{a}_k + \frac{\partial \mathbf{S}}{\partial g_j}\frac{\partial \mathbf{a}_k}{\partial g_l} + \frac{\partial \mathbf{S}}{\partial g_l}\frac{\partial \mathbf{a}_k}{\partial g_j} + \mathbf{S}\frac{\partial^2 \mathbf{a}_k}{\partial g_l \partial g_j} \quad (16)$$

and

$$\frac{\partial^2 \mathbf{a}_{k+1}}{\partial g_l \partial g_j} = \mathbf{C}\frac{\partial^2 \mathbf{b}_k}{\partial g_l \partial g_j} + \mathbf{C'}\frac{\partial^2 \mathbf{b}_{k-1}}{\partial g_l \partial g_j} \quad (17)$$

The advantage of the TLM routine is that it calculates the sensitivities simultaneously in the same iteration process as the calculation of the output response.

4.0 EXAMPLE

A three adaptor WDF with the series configuration is designed and optimized in the time domain. Fig. 5 shows the specified impulse response along with an initial response. The delay T_d is 0.1 msec.

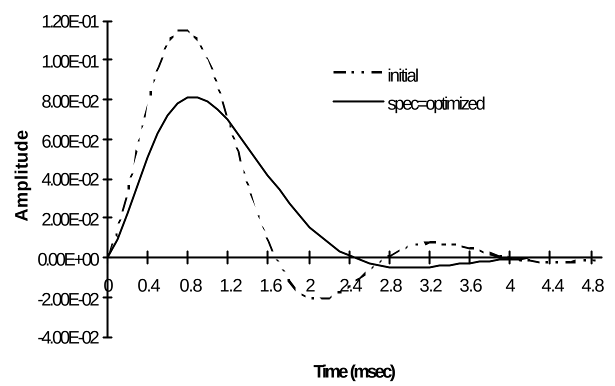

Fig. 5: Optimization of a Ladder WDF Using TLM

The initial impulse response as shown in the figure can be shown to correspond to the following vector:
g=[0.180196, 0.017839, 0.017181, 0.045470, 0.523834, 1.0].

The specification is met with 23 function evaluations using least 4th optimization [11]. The algorithm is simulated in the MATLAB environment to advantageously execute the extensive matrix calculations involved. The resulting coefficient vector after the optimization procedure is g=[0.229713, 0.009625, 0.009113, 0.115408, 0.530617, 1.0].

5.0 CONCLUSIONS

This paper has shown a time domain approach to ladder WDF design. It extends the design approach of using the uni-dimensional transmission line network to represent cascaded unit element WDFs, to the two-dimensional model required to represent ladder WDFs. The design parameters of WDFs are the multiplying coefficients, while for conventional transmission line networks the design parameters are the characteristic impedance of the transmission lines. By using the TLM algorithm, the calculations of the sensitivities of the impulse response with respect to the design parameters are performed in the same iteration process as calculating the impulse response itself. This suggests the possibility of automated ladder WDF design.

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BIOGRAPHY

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